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Review

Price bounds of mortality-linked security in incomplete insurance market

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ABSTRACT

This study investigates reasonable price bounds for mortality-linked securities when the issuer has only a partial hedging ability. The price bounds are established by minimizing the difference between the benchmark price and the replicating portfolio cost subject to the gain–loss ratio of excess payoff of the mortality-linked securities. In contrast to the previous studies, the assumptions of no-arbitrage pricing and utility-based pricing are not fully employed in this study because of the incompleteness of the insurance securitization market. Instead, a framework including three insurance basis assets is constructed to search for the price bounds of mortality-linked securities and use the Swiss Re mortality catastrophe bond, issued in 2003, as a numerical example. The proposed price bounds are valuable for setting bid–asked spreads and coupon premiums, and establishing trading strategies in the raising mortality securitization markets.

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1. Introduction

In recent years, the phenomena of longevity and mortality catastrophe have increased business risk of the insurance and reinsurance industry. Mortality securitization or mortality-linked securities (hereafter, MLSs) are regarded as a prescription to mitigate these risks. By issuing MLSs, insurers can transfer their motility-sensitive exposures to a vested number of investors in the capital market by compensating them with a reasonable risk premium.

The MLS market grew rapidly after two seminal innovations: the Swiss Re mortality bond issued in 2003 and the EIB/BNP longevity bond issued in 2004 (Blake et al., 2006a; Lane, 2011). Because of these innovations, the valuation approaches on MLSs have become a major concern in the literature. We classify the approaches into four types. (a) The risk-adjusted process or no-arbitrage pricing: for example, Cairns et al. (2006b) develop a two-factor mortality process and calibrate it to the EIB/BNP longevity bond price to obtain the implied market price of longevity risks.[1] (b) The Wang transform: the approach by Wang (2000, 2002) provides a distortion operator that transforms the underlying distribution into a risk-adjusted distribution, and the MLS price equals the expected value under the risk-adjusted probability discounted by risk-free rate.[2] (c) Instantaneous Sharpe ratio: Milevsky et al. (2005) propose that the expected return on the MLS equals the risk-free rate plus the Sharpe ratio times its standard deviation.[3] (d) The utility-based valuation: the utility-based method specifies an investor's utility function and maximizes the agent's expected utility subject to wealth constraints to obtain the MLS equilibrium price.[4]

Whereas the aforementioned methods are useful in certain applications, these approaches are limited for MLS valuation because they either require price information or presume specific utility functions. For example, the no-arbitrage pricing method requires a transaction price to obtain the market prices of mortality or longevity risk. However, transaction prices are typically unavailable because most MLSs are traded in an over-the-counter market. For another example, the utility-based method assigns a specific utility function a priori to derive the MLS price. Such a presumption on utility function may cause a high modeling risk and specification errors. To circumvent these limitations, we proposed a pricing model that can incorporate the modeling risk and exclude the arbitrage opportunities at the same time.

Another feature of the MLS market is its incompleteness that results from the underlying mortality rates, which are usually untradeable in financial markets. Therefore, to dynamically or perfectly replicate the payoffs of MLSs by using traditional life and annuity contracts is impossible. In other words, in an incomplete MLS market, the no-arbitrage pricing method can only provide a price range or a *price bound*, instead of a single value. Furthermore, the price bound implied by the classical no-arbitrage pricing rule is too wide to be used. Thus, tightening the no-arbitrage price bound is appealing. To our knowledge, only a few studies deal with the imperfect hedging ability of the MLS issuers.

This article seeks reasonable price bounds for MLSs in an incomplete market and uses the Swiss Re mortality catastrophe bond as an example. To accomplish this, the gain–loss ratio of Bawa and Lindenberg (1977) and Bernardo and Ledoit (2000) is adapted to impose a subjective constraint on the potential payoff of MLSs. Intuitively, a reasonable MLS price should not imply an excessively high or an excessively low gain–loss ratio. We use this idea to impose constraints on the MLS prices. The proposed gain–loss bound contributes to the literature in the following aspects. First, the model misspecification and model risk, such as individual utility and mortality process misspecification, can be considered in the model by replacing a single price with a price bound. The extreme assumptions of the utility-based and the no-arbitrage method are avoided. Second, the price bound can be derived from the utility-based or the no-arbitrage method, indicating that the MLS transaction data are unnecessary. This is helpful in analyzing MLS prices when we possess a handful of data. Third, the bounds derived by the gain–loss ratio method exhibit the same rationale as the good-deal bounds derived using the Sharpe ratio method (Cochrane and Saa-Requejo, 2000). Fourth, the gain–loss ratio is applicable to the existing MLS valuation approaches; it can be regarded as an add-on analysis tool for MLS prices. Finally, the price bound is useful for setting bid–asked spreads, risk premiums, and establishing trading strategies for MLS in practice.

The remainder of this article is organized as follows: the valuation methods are set into a two-period economy and the intuitions of the price bounds are explained in Section 2, the general gain–loss bound model is derived with multiple basis assets and states in Section 3, a brief discussion on the Swiss Re mortality bond and the construction of price bounds by solving two optimization problems are included in Section 4, the numerical results of price bounds, using the mortality process by Lin and Cox (2008) and Chen and Cox (2009), are presented in Section 5, and the conclusion and limitations are presented in Section 5.

2. An example of mortality-linked security price bounds

We first illustrate the basic assumptions of the insurance securitization market and then provide an example to derive the gain–loss price bound. The generalized gain–loss bound with multiple basis assets in J states is introduced in Section 3.

2.1. Assumptions regarding the mortality-linked security market

To render MLS securitization and valuation compatible with the traditional insurance market, several assumptions have to be made firstly.

1. The market is a two-period economy. The issuer forms his or her hedging (replicating) portfolio with pure life insurances, pure

[1] Biffis (2005), Biffis and Millossovich (2006), Chen and Cox (2009), Dahl and Møller (2006), Hainaut and Devolder (2008) and Wills and Sherris (2010) also use this method to find the fair value of the MLS.

[2] Dowd et al. (2006), Lin and Cox (2008), Denuit et al. (2007) and Cox et al. (2006) use this approach to price the MLS.

[3] Please see Bayraktar and Young (2007), Young (2008) and Bayraktar et al. (2009).

[4] Please refer Hainaut and Devolder (2008), Dahl and Møller (2006), Cox et al. (2010) and Tsai and Tzeng (2013).