Insurance: Mathematics and Economics 55 (2014) 10–17

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Prediction in a non-homogeneous Poisson cluster model

Muneya Matsui *

Department of Business Administration, Nanzan University, 18 Yamazato-cho Showa-ku Nagoya, 466-8673, Japan

HIGHLIGHTS

- A non-homogeneous Poisson cluster model is studied, motivated by insurance applications.
- The cluster center process is Poisson and the cluster member process is an additive process.
- Given the past observations we consider the expected values of future increments and their MSE.
- The proposed process can cope with non-homogeneous observations.

ARTICLE INFO

Article history:
Received July 2013
Received in revised form November 2013
Accepted 1 December 2013

MSC:
primary 60K30
secondary 60G25
60G55

Keywords:
Poisson cluster model
Prediction
Conditional expectation
Claims reserving
Insurance
Lévy process
Shot noise

ABSTRACT

A non-homogeneous Poisson cluster model is studied, motivated by insurance applications. The Poisson center process which expresses arrival times of claims, triggers off cluster member processes which correspond to number or amount of payments. The cluster member process is an additive process. Given the past observations of the process we consider expected values of future increments and their mean squared errors, aiming at application in claims reserving problems. Our proposed process can cope with non-homogeneous observations such as the seasonality of claims arrival or the reducing property of payment processes, which are unavailable in the former models where both center and member processes are time homogeneous. Hence results presented in this paper are significant extensions toward applications.

1. Introduction

A cluster point process is one of the most important classes of point processes, and has two driving processes, the process of the cluster center and the process of each cluster (see e.g. Daley and Vere-Jones, 1988 or Westcott, 1971). The Poisson cluster process is a version of cluster point processes whose center process is a Poisson process. The process has been applied to a wide range of different fields such as earthquake aftershocks (Vere-Jones, 1970), motor traffic (Bartlett, 1963), computer failure times (Lewis, 1964) and broadband traffic (Hohn et al., 2003) to name just a few. For more on applications we refer to Daley and Vere-Jones (1988).

In this paper motivated by insurance applications we investigate the Poisson cluster process of the form,

$$M(t) = \sum_{j=1}^{N(1)} L_j(t - T_j), \quad t \geq 1, \tag{1.1}$$

where $0 < T_1 < T_2 < \cdots$ are points of non-homogeneous Poisson (NP for short) processes $N(t)$ and (L_j), $j = 1, 2, \ldots$ are an i.i.d. sequence of additive processes with $L_j(t) = 0$, a.s. for $t \leq 0$, such that (T_j) and (L_j) are independent. In the insurance context $T_j \leq 1$ would be the arrival of claims within a year, and $(L_j(t - T_j))_{j:T_j \leq 1}$ are the corresponding payment processes from an insurance company to policyholders. We could also regard the cluster as the counting process of payment number. Hence $M(t)$ would be the total number or amount of payments for the claims arriving in a year and being paid in the interval $[0, t]$, $t \geq 1$. Historically this kind of stochastic process modeling goes back to Lundberg (1903) (see comments in Mikosch, 2009, p. 224) who introduced the Poisson process for a simple claim counting process.

* Tel.: +81 52 832 3310.
E-mail addresses: mmuneya@gmail.com, mmuneya@nanzan-u.ac.jp.

0167-6687/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.insmatheco.2013.12.001

Norberg (1993, 1999) has been considered to give publicity to the point process approach in a non-life insurance context.

Our focus in this paper is on the prediction of the future increments

$$M(t, t+s] = M(t+s) - M(t), \quad t \geq 1,\ s > 0,$$

for some suitable σ-fields $\mathcal{F}_t$ i.e. we will calculate $E[M(t, t+s] \mid \mathcal{F}_t]$ and evaluate the mean squared error of the prediction. These kinds of problems are known to be claims reserving problems, which have been intensively studied from old times.

In reference to prediction problems with the model (1.1), Mikosch (2009) introduced the model into the claims reserving problems with a simple settings such that both the center process and clusters are homogeneous Poisson processes, where a numerically tractable form of the predictor $E[M(t, t+s] \mid M(t)]$ is also obtained. More generally, Matsui and Mikosch (2010) consider Lévy or truncated compound Poisson for clusters and obtain analytic forms of both prediction and its mean squared error. Matsui (2013a) introduced a variation of the model (1.1) which starts randomly given the number of cluster processes at each jump point of the underlying process $N(t)$ and also obtain predictors and their errors. In a different path Jessen et al. (2011) takes simpler but useful point process modeling for the problem. See also Rolski and Tomanek (2011) which investigates asymptotics of conditional moments arising from prediction problems. Notice that almost all processes used in the context are included in the class of Lévy processes, which implies that increments are time homogeneous.

In this paper we introduce non-homogeneity into both underlying Poisson process $N(t)$ and clusters L_j by the use of additive processes such that the processes have independent but not always stationary increments. More precisely, we assume an NP process for N, whereas each cluster L_j is assumed to be an additive process which is given by a certain integral of a general Poisson random measure. Our intention here is to model the seasonality of claims arrivals and the curved line of payment numbers or amounts which are naturally observed from data (see e.g. Table 2 of Jessen et al., 2011). Again we emphasize that in the former models (Mikosch, 2009; Matsui and Mikosch, 2010) or (Matsui, 2013a), they intensively use Lévy clusters which are the processes of stationary independent increments and therefore are time homogeneous.

This paper is organized as follows. In Section 2, we consider the model with additive Lévy processes and obtain the conditional characteristic function (ch.f. for short) $E[e^{ixM(t,t+s]} \mid M(t)]$. Based on the derived ch.f. we investigate expressions of $E[M(t, t+s] \mid M(t)]$ where NP clusters and non-homogeneous negative binomial clusters are considered. In both cases, we derive a recursive algorithm to calculate exact values of predictors and their conditional mean squared errors. In Section 3 the prediction $E[M(t, t+s] \mid \mathcal{F}_t]$ with different σ-fields $\mathcal{F}_t$ is investigated where we notice the delay in reporting times of claims and consider the number of reported claims until time t for $\mathcal{F}_t$. Exact analytic forms for both predictors and their mean squared errors are calculated.

Finally, we briefly explain the basics of an additive process $\{L(t)\}_{t\geq 0}$ based on Sato (1999, p. 53). It is well known that the process is stochastically continuous, and has independent increments with càdlàg path starting at $L(0) = 0$ a.s. The distribution of the process $\{L(t)\}_{t\geq 0}$ at time t is determined by its generating triplet (A_t, ν_t, γ_t) since this determines the corresponding ch.f. Among additive processes we work on the process of the so called jump part such that the distribution of time t is given by the inversion of

$$E[e^{ixL(t)}] = \exp\left\{\int_{(0,t]\times\mathbb{R}} (e^{ixv} - 1)\nu(d(u, v))\right\}, \tag{1.2}$$

where a measure ν on $(0, \infty) \times \mathbb{R}$ satisfies $\nu((0, \cdot] \times \{0\}) = 0$, $\nu(\{t\} \times \mathbb{R}) = 0$ and

$$\int_{(0,t]\times\mathbb{R}} (|u| \wedge 1)\nu(d(u, v)) < \infty, \quad \text{for } t \geq 0.$$

In this case the generating triplet is $(0, \nu_t, 0)$ with $\nu_t(B) := \nu((0, t] \times B)$ for any Borel set $B \in \mathcal{B}(\mathbb{R})$. The first condition means $\nu_t(\{0\}) = 0$ and the second one implies stochastic continuity, whereas the third controls smoothness of the path. In view of (1.2) one sees that an additive process has an integral representation by a Poisson random measure on $(0, \infty) \times \mathbb{R}$ with intensity measure ν. We refer to Theorems 19.2 and 19.3 of Sato (1999) for the jump part of an additive process. Although we could treat a more general additive process by including the continuous part or another version of the jump part, the prediction procedure would be more complicated and we confine the process of the cluster as such.

2. Prediction in Poisson cluster model

In this section firstly we give general prediction results which are valid for all additive Lévy clusters given by (1.2) and then we investigate numerically tractable expressions with examples. More precisely, we study expressions of the conditional expectation of $M(t, t+s]$ given $M(t)$, $t \geq 1$, $s > 0$ and its mean squared error.

The following is basic for the model (1.1).

Lemma 2.1. *Assume the model (1.1) with i.i.d. additive processes L_k, $k = 1, 2, \ldots$ and an NP process N with mean measure Λ such that $\Lambda[0, \infty) < \infty$. We write the generic form of processes L_k as L. Then the ch.f. is given by*

$$E[e^{ixM(t)}] = \exp\left\{\int_{[0,1]} (E[e^{ixL(t-u)}] - 1)\Lambda(du)\right\}$$

for $t \geq 1$ and $x \in \mathbb{R}$. Moreover, assume that $E[L(t)]$ finitely exists for all $t \geq 0$, then

$$E[M(t)] = \int_{[0,1]} E[L(t-u)]\Lambda(du), \quad t \geq 1.$$

Assume that $E[L^2(t)]$ is finite for all $t \geq 0$. Then, for $1 \leq s \leq t$,

$$\mathrm{Cov}(M(s), M(t)) = \int_{[0,1]} (E[L^2(s-u)] + E[L(s-u)] \times E[L(s-u, t-u]])\Lambda(du).$$

We are starting to observe the conditional ch.f. of $M(t, t+s]$ given $M(t)$.

Lemma 2.2. *Assume the model (1.1) with i.i.d. additive processes L_k, $k = 1, 2, \ldots$ given by (1.2) and an NP process N with the mean value function $\Lambda(\cdot)$. For $m = 1, 2, \ldots, s > 0$, $t \geq 1$ and $x \in \mathbb{R}$, the conditional ch.f. of $M(t, t+s]$ given $\{M(t) \in A\}$ for any Borel set A has the form given in (2.1) of Box I for an i.i.d. sequence (V_j) with density function*

$$F(dx) = \Lambda(dx)/\Lambda(1), \quad 0 \leq x \leq 1 \tag{2.2}$$

such that (V_j), (L_j) and N are mutually independent.