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Combining chain-ladder claims reserving with fuzzy numbers

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ABSTRACT

In this paper we extend the classical chain-ladder claims reserving method using fuzzy methods. Therefore, we derive new estimators for the claims development factors as well as new predictors for the ultimate claims. The advantage in using fuzzy numbers lies in the fact that the model uncertainty is directly included in and can be controlled by the "new" fuzzy claims development factors. We also provide an estimator for the uncertainty of the ultimate claims for single accident years and for aggregated accident years.

1. Introduction

1.1. Motivation

Non-life actuaries in insurance companies are faced with the problem that reserves for outstanding claims need to be predicted in an appropriate way. There exist a number of purely computational and stochastic models. One of the methods widely used in practice is the chain-ladder (CL) method (cf. Wüthrich and Merz, 2008 and Mack, 1993). Even though the development factors calculated in the CL model are crisp, actuaries tend to adjust the factors afterwards due to their subjective judgement. Thus, if the calculated, i.e. crisp value of the CL-factor is c, the adapted value can be perceived as approximately c. In this way, vagueness is added to the CL model. We propose a model in which the CL factors do not need to be adjusted retrospectively and, nevertheless, are flexible and include uncertainty. Hence, our method focuses on situations in which actuaries tend to adjust development factors and therefore vagueness is on hand.

Fuzzy Set Theory (FST) as introduced in Zadeh (1965) has been used for many different applications in insurance. It all started off with the work of de Wit (1982) in which FST is applied to underwriting. A similar approach is done in Lemaire (1990).

A survey of applications can be found in Shapiro (2004). Previously stated articles can be classified as insurance or actuarial science. However, the field of claims reserving within actuarial science has not received much attention in context with fuzzy sets. De Andrés Sánchez and Terceño Gómez (2003) propose an application of fuzzy regression (FR) to the London chain-ladder Method by Benjamin and Eagles (1986) in order to determine Incurred But Not Reported (IBNR) reserves. They make use of FR as described in Tanaka and Ishibuchi (1992).

De Andrés Sánchez (2006) suggests a method for claims reserving which applies a FR technique by Ishibuchi and Nii (2001) to a claims reserving scheme proposed by Sherman (1984). De Andrés Sánchez (2007) combines FR with Taylor's geometric separation method as described in Taylor (1978). Başer and Apaydin (2010) apply hybrid fuzzy least-squares regression analysis as suggested by Chang (2001) to the London chain-ladder Method. A recent work by De Andrés Sánchez (2012) applies FR to a claims reserving method suggested by Kremer (1982).

The articles dealing with claims reserving all have in common that they use FR to obtain the reserves. In this paper we enhance the classical CL method by using fuzzy numbers (FNs) and fuzzy arithmetic. We obtain our results by using triangular FNs (TFNs) which have been introduced as a special case of L–R FNs by Dubois and Prade (cf. Dubois and Prade, 1978 or Dubois and Prade, 1979). By doing so no information especially regarding the uncertainty of a quantity is lost. In stochastic considerations often expected

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values, i.e. real numbers, of random variables are regarded whereas in a fuzzy environment FNs which assign every real number a grade of membership are contemplated.

When using TFNs to model the CL-factors the mode as well as the left and right spreads need to be estimated. In this paper we present one possible estimator and conduct the analysis with this choice.

The structure of the paper is as follows. In the next section the classical CL method, fuzzy numbers and fuzzy arithmetic are introduced. In Section 2 we present the fuzzy chain-ladder (FCL) model. In Section 3 we describe how to obtain the claims reserves. Section 4 is dealing with the model uncertainty using a measurement of uncertainty proposed by Pal and Bezdek (1994). In Section 5 an example is presented. The article ends with a conclusion.

1.2. The chain-ladder method

The CL method is one of the widely used claims reserving methods in practice due to its simplicity and nonetheless good results. It assumes that the increase of the cumulative claims from one development year to another acts on average like in the previous accident years.

A claim occurs in an accident year. Since not all claims are reported immediately and are settled instantly as future demands can arise over time cumulative claims can change over several development years. In the following we denote with $C_{i,j}$ cumulative claims made in relative accident year $i \in \{0, \ldots, I\}$ and relative development year $j \in \{0, \ldots, J\}$. We assume that we are at time (calendar year) $t = I$, i.e. we have the following set of given observations:

$$\mathcal{D}_I = \{C_{i,j} \mid i + j \leq I\}. \tag{1.1}$$

Fig. 1 demonstrates the specifications made above. The upper left part in the given development triangle is observable at time $t = I$, while the lower right part is unobservable. For simplification we only consider the case $I = J$, i.e. development triangles. Of course all formulas also hold true for development trapezoids, i.e. $I > J$. In the following we assume that claims are settled after J years.

The CL method's objective is to fill the development triangle, especially calculate the ultimate claims. Mack (1993) proposed the following Model Assumptions.

Model Assumptions 1.1 (*Distribution-Free Chain Ladder*)**.** We assume for the cumulative claims $C_{i,j}$:

(a) Cumulative claims $C_{i,j}$ of different accident years i are independent.
(b) There exist development factors $f_0, \ldots, f_{J-1} \in \mathbb{R}^+$ and parameters $\sigma_0^2, \ldots, \sigma_{J-1}^2$ such that

$$E\left[C_{i,j+1} \mid C_{i,j}\right] = f_j C_{i,j} \tag{1.2}$$

$$\mathrm{Var}(C_{i,j+1} \mid C_{i,j}) = \sigma_j^2 C_{i,j} \tag{1.3}$$

hold true for all $i \in \{0, \ldots, I\}$ and $j \in \{1, \ldots, J\}$.

Since the parameters are often not observable in practice they need to be estimated. This is done with so called claims development factors (or CL-factors):

$$\hat{f}_j = \frac{\sum_{i=0}^{I-j-1} C_{i,j+1}}{\sum_{i=0}^{I-j-1} C_{i,j}}, \quad j = 0, \ldots, J-1.$$

We yield the full triangle and the ultimate claims by successively multiplying the diagonal elements with the CL factors, i.e. $C_{I-i,i} \cdot \hat{f}_{I-i} \cdot \ldots \cdot \hat{f}_{J-1}$.

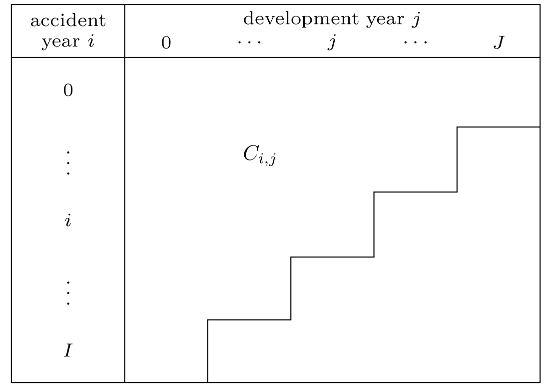

Fig. 1. Development triangle at time $t = I$ with observable cumulative claims $C_{i,j}$ in the upper left part, while the lower right part is unobservable.

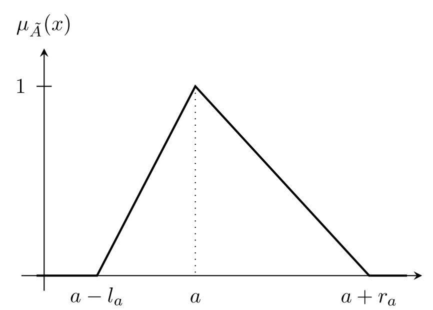

Fig. 2. A TFN $\tilde{A} = (a, l_a, r_a)$.

1.3. Fuzzy numbers and fuzzy arithmetic

In our fuzzy chain-ladder (FCL) model we describe the unobservable chain-ladder development factors introduced in Section 2 as FNs. To define these FNs we have to define fuzzy subsets in advance.

Definition 1.2 (*Fuzzy Subsets*)**.** A fuzzy subset $\tilde{A}$ over $\mathbb{R}$ is defined as

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in \mathbb{R}\} \tag{1.4}$$

with membership function $\mu_{\tilde{A}}$ given by:

$$\mu_{\tilde{A}} : \mathbb{R} \to [0, 1]. \tag{1.5}$$

Remarks 1.3.

- The membership function $\mu_{\tilde{A}}$ describes to what extent an element $x \in \mathbb{R}$ belongs to the fuzzy set $\tilde{A}$.
- The fuzzy subset $\tilde{A}$ is called "normal" if and only if there exists an $x \in \mathbb{R}$ with $\mu_{\tilde{A}}(x) = 1$.
- From a statistical point of view the membership function $\mu_{\tilde{A}}$ plays a similar role as a density function of a continuous random variable X.

Fig. 2 is an example of a (triangular) membership function $\mu_{\tilde{A}}$.

In the following we are only working with triangular membership functions and the corresponding triangular fuzzy numbers (TFNs). The reason for using TFNs is given by the fact that they are easy to handle and can be interpreted intuitively.

Definition 1.4 (*Triangular Fuzzy Numbers (TFNs)*)**.** A triangular fuzzy number is a fuzzy subset over $\mathbb{R}$ with membership function