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Optimal surrender policy for variable annuity guarantees

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ABSTRACT

This paper proposes a technique to derive the optimal surrender strategy for a variable annuity (VA) as a function of the underlying fund value. This approach is based on splitting the value of the VA into a European part and an early exercise premium following the work of Kim and Yu (1996) and Carr et al. (1992). The technique is first applied to the simplest VA with GMAB (path-independent benefits) and is then shown to be possibly generalized to the case when benefits are path-dependent. Fees are paid continuously as a fixed percentage of the fund value. Our approach is useful to investigate the impact of path-dependent benefits on surrender incentives.

1. Introduction

A variable annuity (VA) is a unit-linked insurance product offering a variety of financial guarantees. Usually the policyholder pays an initial premium to initiate the contract. This premium is invested in a mutual fund selected by the policyholder. Many types of guarantees and options can be added to the contract (for more details, see Hardy, 2003). In this paper we will focus on a variable annuity contract that guarantees a minimum amount at maturity. This type of VA is referred to as a guaranteed minimum accumulation benefit (GMAB) (see Bauer et al., 2008). We study two cases, one with a point-to-point guarantee linked to the terminal value of the fund and a guarantee linked to the average value of the fund.

In most cases, the policyholder can choose to lapse the VA contract and receive a surrender benefit, which is less than or equal to the value accumulated in the underlying account. For example Kling et al. (2011) show that unexpected lapses represent a significant risk for the insurer. In fact, selling a VA contract is expensive and insurers typically reimburse the expense incurred using the fees paid during the first years of the policy. If the policyholder lapses before the initial expenses are reimbursed, the insurer may experience a loss. Even if they occur later during the life of the contract, lapses can be very expensive.

For this reason, the option to lapse the contract needs to be taken into account and priced in the contract. This is not necessarily simple since assumptions must be made on the surrender behavior of policyholders. Different approaches have been taken in the literature, ranging from a simple deterministic surrender rate to more sophisticated models, like De Giovanni (2010)'s rational expectation and Li and Szimayer (2010)'s limited rationality. Most of these approaches assume that the policyholder cannot calculate the exact risk-neutral value of the contract, and that he may be influenced by exogenous factors.

Another way to approach the surrender problem is to assume the policyholder is perfectly rational and will surrender their contract only when it is optimal to do so from a financial perspective. In this approach, the surrender option is analogous to an American option that can be surrendered at any time before maturity (see Grosen and Jørgensen, 2000). Assuming that the policyholder is perfectly rational leads to an upper bound for the price of the surrender option and gives a lot of insight on the intrinsic value of the options in the VA contract. Although it is not necessarily used to obtain the final price of the VA contract, it can be very useful to assess the risk of the optimal surrender. Furthermore, while there are also other reasons why a VA contract might be surrendered, some policyholders tend to act in a rational way. In their study, Knoller et al. (2011) investigate various hypotheses for an early surrender. For one, they analyze the moneyness of the option as a reason to lapse the VA. This is similar to optimally surrendering the contract when the maturity benefit is out-of-the-money. They find that financial literacy leads to a higher sensitivity towards the moneyness. They also examine other reasons for lapsing the contract such as financial needs of the policyholder or better opportunity costs in times of rising market interest rates.

While it is common practice for insurers to charge a constant fee rate as a percentage of the fund value to cover the maturity benefit and other financial guarantees, many authors assume that all the

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fees are included in the initial premium (see, for example, Grosen and Jørgensen, 2002, Bacinello, 2003a,b, Siu, 2005, Bacinello et al., 2009, 2010, Bernard and Lemieux, 2008). However, as pointed out for instance by Bauer et al. (2008), Milevsky and Salisbury (2001) and Bernard et al. (2013), a fee paid as a regular constant percentage of the fund might increase the incentive to surrender the contract before the maturity if the fund value is high. This is due to the mismatch between the amount of the fee and the value of the guarantee option. When the fund value is high, the guarantee at maturity is deep out-of-the-money and it is unlikely that the policyholder will make use of the option at expiration. However, since the fee is charged as a percentage of the fund value, the amount of the fee is large. This mismatch represents an incentive to surrender the VA contract and should be taken into account especially when the policyholder is assumed to lapse optimally. Milevsky and Salisbury (2001) discuss this issue, and argue that surrender charges are necessary to hedge VA contract appropriately. In fact, in most VA contracts sold in the industry, early surrenders trigger a surrender charge and the policyholder does not receive the full value accumulated in the underlying fund. This is especially true in the first years of the contract. This surrender charge has many purposes, one of which is to reduce the incentive to surrender. It is also in place to recover the high expenses related to the sale of the VA contract. While this fee does give the policyholder an incentive to remain in the contract, there are many situations where it is optimal to surrender, even after taking the surrender charge into account.

In this paper we investigate the optimal surrender strategy for a variable annuity contract with constant fee rate paid as a percentage of the fund and a GMAB feature. We first consider a simple point-to-point guarantee and derive an integral representation for the price of the contract, which can be solved to compute the optimal surrender boundary. To do so, we use no-arbitrage arguments presented, among others, by Kim and Yu (1996) and Carr et al. (1992). This technique originally designed for vanilla call options can be extended to more complex path-dependent payoffs linked for example to the average. Our objective is to illustrate a general technique to compute the optimal surrender strategy for a possibly path-dependent contract. This technique may help to understand the effect of complex path-dependent benefits on surrender incentives and could be useful to reduce the surrender option value by modifying the type of benefits offered and justifying the need for path-dependent benefits. This is in contrast with the recent proposal by Bernard et al. (2013) and Bernard and MacKay (2014) to influence the surrender behavior by charging a state-dependent fee structure, instead of charging a constant fee rate.[1]

The paper is organized as follows. In Section 2 we state the setting. The optimal surrender policy is derived in Section 3. Section 4 extends this method to path-dependent payoffs. In Section 5 we apply these results to numerical examples and analyze the sensitivity of the boundary with respect to a range of parameters. Section 6 concludes.

2. Setting

Consider a variable annuity contract with a guaranteed minimum accumulation benefit G_T at maturity T. This accumulation benefit is computed as $G_T = Ge^{gT}$ where the guaranteed rate g satisfies $g < r$. Let F_t denote the underlying accumulated fund value of the variable annuity at time t. We assume that the insurance company charges a constant fee c for the guarantee, which is continuously withdrawn from the accumulated fund value F_t. Furthermore, we assume that the policyholder pays a single premium to initiate the contract. The insurer then invests this premium in the fund or index that was chosen by the policyholder. We denote this underlying fund or index by S_t and assume that it follows a geometric Brownian motion. Therefore, its dynamics under the risk-neutral measure $\mathbb{Q}$ are given by

$$dS_t = rS_t dt + \sigma S_t dW_t, \tag{1}$$

where r is the risk-free interest rate, $\sigma > 0$ the constant volatility and W_t the Brownian motion. We denote by $\mathcal{F}_t$ the natural filtration associated with this Brownian motion. In this case, the stock price at time $u > t$ given the stock price at time t has a lognormal distribution and is explicitly given by

$$S_u = S_t e^{\left(r-\frac{\sigma^2}{2}\right)(u-t)+\sigma(W_u-W_t)}.$$

In this paper, we are only concerned with the pricing of the surrender option and as such, we can treat the whole problem under the risk-neutral measure. This choice is also motivated by the use of no-arbitrage arguments in the derivation of the expression for the surrender option. It is based on the assumption that investors optimize over all possible surrender strategies and will choose to surrender optimally from a financial perspective. As investors do not always act optimally, our derivations lead to an upper bound on the price of the surrender option.

The following results (2) and (3) will be useful to derive the results of this paper. Since the insurance company continuously takes out a percentage fee c of the fund value, we have the following relationship between S_u and F_u at any time u

$$F_u = e^{-cu}S_u = F_t e^{\left(r-c-\frac{\sigma^2}{2}\right)(u-t)+\sigma(W_u-W_t)}. \tag{2}$$

Therefore, the conditional distribution of $F_u|F_t$ for $u > t$ is a lognormal distribution with log-scale parameter $\ln(F_t)+(r-c-\frac{\sigma^2}{2})(u-t)$ and shape parameter $\sigma^2(u-t)$. Hence, the risk-neutral transition density function of F_u at time $u > t$ given F_t equates to

$$f_{F_u}(x|F_t) = \frac{1}{\sqrt{2\pi\sigma^2(u-t)}x} e^{-\frac{\left[\ln\left(\frac{x}{F_t}\right)-\left(r-c-\frac{\sigma^2}{2}\right)(u-t)\right]^2}{2\sigma^2(u-t)}}, \quad x > 0. \tag{3}$$

Note that in this paper we restrain ourselves to the case when the underlying follows a geometric Brownian motion, which presents a simple closed expression for its transition density. However, the method we present here can easily be extended to more general market models. We discuss this point briefly in the conclusion.

2.1. Fair fee for the European benefit

Let us assume in this paragraph that the VA cannot be surrendered early and let c be the fee charged by the insurer between 0 and T. Note that the fund value at time T depends on this fee. We denote by F_T^c the value at T of the fund given that the fee charged during $[0, T]$ is equal to c and by $\phi(F_\bullet^c, T)$ the payoff at maturity T which may depend on the path of the fund denoted by $F_\bullet^c$. If the fee c is fair (for the European benefit), we denote it by c^* and it fulfills

$$F_0 = \mathbb{E}[e^{-rT}\max(\phi(F_\bullet^{c^*}, T), G_T)], \tag{4}$$

where F_0 is the lump sum paid initially by the policyholder net of initial expenses and management fees. This fee c^* exists and is unique. To compute this fair fee, it is always possible to use Monte Carlo techniques. However when the distribution of $\phi(F_\bullet^c, T)$ is known, an analytical formula may be derived, which subsequently can be solved for c^*. For example when $\{X_t\}_{t\in[0,T]}$ is a Markov

[1] In the model of Bernard et al. (2013) the policyholder only pays the fee as long as the fund value stays underneath a certain barrier. Bernard and MacKay (2014) investigate periodic fees set at inception as a fixed, deterministic amount to pay for the guarantees.