Insurance: Mathematics and Economics 55 (2014) 167–179

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Dependent interest and transition rates in life insurance

Kristian Buchardt *

Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen O, Denmark
PFA Pension, Sundkrogsgade 4, DK-2100 Copenhagen O, Denmark

HIGHLIGHTS

- Introduction of Dependent Forward Rates.
- Review of mathematical methods for dependent interest and/or transition rates.
- Review of different forward rate definitions.
- Surrender modelling with analytic methods and interest rate dependence.
- Solvency II capital requirement with policyholder modelling.

ARTICLE INFO

Article history:
Received January 2013
Received in revised form December 2013
Accepted 7 January 2014

JEL classification:
G22

Keywords:
Affine processes
Doubly stochastic process
Multi-state life insurance models
Policyholder behaviour
Solvency II
Surrender

ABSTRACT

For market consistent life insurance liabilities modelled with a multi-state Markov chain, it is of importance to consider the interest and transition rates as stochastic processes, for example in order to consider hedging possibilities of the risks, and for risk measurement. In the literature, this is usually done with an assumption of independence between the interest and transition rates. In this paper, it is shown how to valuate life insurance liabilities using affine processes for modelling dependent interest and transition rates. This approach leads to the introduction of so-called dependent forward rates. We propose a specific model for surrender modelling, and within this model the dependent forward rates are calculated, and the market value and the Solvency II capital requirement are examined for a simple savings contract.

1. Introduction

Life insurance liabilities are traditionally modelled by a finite state Markov chain with deterministic interest and transition rates. In order to give a market consistent best estimate of the present value of future payments, it has become of increasing interest to let the interest and transition rates be modelled as stochastic processes. The stochastic modelling is important in order to consider hedging possibilities of the risks. With the Solvency II rules, stochastic modelling of the interest and transition rates is also important from a risk management perspective. Modelling the interest and transition rates as stochastic processes is traditionally done with an independence assumption. In this paper, we relax the independence assumption, and consider basic valuation with dependence between the interest and one or more transition rates. This is done with continuous affine processes for the modelling of the dependent rates. The study of valuation of life insurance liabilities with dependent rates leads to the definition of so-called dependent forward rates. These are natural quantities that appear in case of dependence, replacing the usual forward rates, which are not directly applicable. Using the theory of dependent affine rates, we consider the case of surrender modelling, and propose a specific model for dependent interest and surrender rates. This is of particular interest from a Solvency II point of view. Within this model, a simple savings contract with a buy-back option is considered. We calculate the dependent forward rates, the market value and the Solvency II capital requirement. This is done in part without hedging, and in part with a simple static hedging strategy. We then examine the effect of correlation between the interest and surrender rates.

The study of valuation of life insurance liabilities with stochastic interest and transition rates has received considerable attention

* Correspondence to: Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen O, Denmark. Tel.: +45 29255855.

E-mail address: buchardt@math.ku.dk.

URL: http://math.ku.dk/~buchardt.

0167-6687/$ – see front matter
http://dx.doi.org/10.1016/j.insmatheco.2014.01.004

during the past decades. Primarily the interest and mortality rates have been modelled as stochastic, which is often done with affine processes. For basic applications of affine processes for valuation of life insurance liabilities, see Biffis (2005). Possibilities of hedging can be considered, which is important for market consistent valuation, and for the study of valuation and hedging of life insurance liabilities with stochastic interest and mortality rates, see Dahl and Møller (2006) and Dahl et al. (2008). Another approach to modelling stochastic interest and mortality is taken in Norberg (2012), where the interest and mortality are modelled within a finite state Markov chain setup. In this paper we extend the study of affine interest and transition rates to the case of dependence. We consider how to valuate life insurance liabilities when the interest and one or more transition rates are modelled as dependent affine processes. This is possible in any decrement/hierarchical Markov chain setup, that is, in Markov chains where, when the process leaves a state, it cannot return. We adopt the theory presented in Buchardt (2012), which is reviewed in Section 2 of this paper. This provides the foundation for the study of multidimensional affine processes in life insurance mathematics. The theory presented in Buchardt (2012) is based partly on a result in Duffie et al. (2000), and partly on general theory for multidimensional affine processes presented in Filipovic (2009).

In the financial literature, the concept of a forward interest rate exists, which is convenient, e.g. for representing zero coupon bond prices. This quantity appears naturally in life insurance mathematics, when the interest rate is modelled as a stochastic process. If one also considers a stochastic mortality, independent of the interest rate, it becomes natural to define a forward mortality rate as well. With these forward rates, the expected present value of the life insurance liabilities has a particularly compelling representation. However, if one introduces dependence between the interest and mortality rates, the forward rates are no longer applicable. In this paper, we introduce so-called dependent forward rates that appear naturally and are applicable for representing the expected present value of the life insurance liabilities in a convenient form, in cases where the usual forward rates are not applicable. In Miltersen and Persson (2005), alternative forward mortality rates are defined in order to handle the case of dependence. In the present paper, we show that one of the forward mortality rates defined in Miltersen and Persson (2005) is in general not well defined. For a general discussion on forward rates, and their usefulness, see Norberg (2010), wherein the case of dependence between the rates is discussed as well. One of the consistency problems with forward rates in the dependent setup that is raised in Norberg (2010) is solved by the proposed dependent forward rates introduced in the present paper. Also, the dependent forward rates introduced here generalise the usual definitions of forward rates, in the sense that when there is independence between the rates, the dependent forward rates equal the usual forward rates.

Modelling policyholder behaviour has become of increasing importance with the proposed Solvency II rules, where one is required to consider any dependence between the economic environment and policyholder behaviour, see Section 3.5 in CEIOPS (2009). The study of surrender behaviour can either be made using a rational approach, where the outset is, that the policyholders surrender the contract if it is rational from some economic viewpoint, which is studied in Steffensen (2002). This seems a bit extreme, given that this behaviour is not seen in practice. Another approach is the intensity approach, where the policyholders surrender randomly, regardless whether or not it is profitable in the current economic environment. This is not a perfect way of modelling either, since if the interest rates decrease a lot, a guarantee given in connection with the life insurance contract motivates the policyholders to keeping the contract. For an overview of some of the approaches, see Møller and Steffensen (2007). In De Giovanni (2010), an attempt is made on coupling the two approaches, using two different surrender rate models if it is rational or irrational, respectively, to surrender. In this paper, we propose another way of coupling the two approaches. We let the surrender rate be positively correlated with the interest rate, thus if the interest rate decreases a lot, the surrender rate also decreases, representing that the guarantee inherent in the life insurance contract is of value to the policyholder.

The Solvency II capital requirement is basically, that "the insurance company must have enough capital, such that the probability of default within the next year is less than 0.5%", representing that a default is a 200-year event. When the insurance company updates its mortality tables, or other transition rate tables, this represents a risk that must be taken into account when putting up the Solvency II capital requirement. Mathematically, this can be done using stochastic rates. For an examination of mortality modelling and the Solvency II capital requirement, see e.g. Börger (2010). For a basic discussion of the mathematical formulation of the Solvency II capital requirement, see e.g. Buchardt (2011). In this paper, we determine the Solvency II capital requirement for the simple savings contract where the interest and surrender rate risk is considered, both in the case of no hedging strategy, and also in the case of a simple strategy where interest rate risk is hedged.

The structure of the paper is as follows. In Section 2, we present basic results on multidimensional continuous affine processes, which provides the foundation for the application of dependent affine processes in life insurance mathematics. In Section 3, we present the general life insurance setup with stochastic interest and transition rates, and in Section 4, we propose the definition of dependent forward rates and compare to the usual forward rate definition. In Section 4.1, we discuss other definitions in the literature of forward rates in a dependent setup, and compare them to the dependent forward rates proposed here. In Section 5, we present a specific model for dependent interest and surrender rates. The model is introduced in Section 5.1. We first discuss how to find the Solvency II capital requirement, which is done in Section 5.3, and a simple hedging strategy for the interest rate risk is presented in Section 5.4. Numerical results are presented in Section 5.5, consisting of the dependent forward rates found, and the market value and Solvency II capital requirement, presented for different levels of correlation.

2. Continuous affine processes

The class of affine processes provides a method for modelling interest and transition rates, with the possibility of adding dependence. In this section, we consider general results about continuous affine processes, which we apply in this paper. For more details on the theory presented in this section, see Buchardt (2012).

Let $\mathbf{X}$ be a d-dimensional affine process, satisfying the stochastic differential equation

$$\mathrm{d}X(t) = (b(t) + \mathcal{B}(t)X(t))\,\mathrm{d}t + \rho(t, X(t))\,\mathrm{d}W(t),$$

where $\mathbf{W}$ is a d-dimensional Brownian motion. Here, $b : \mathbb{R}_+ \to \mathbb{R}^d$ is a vector function, and $\mathcal{B} : \mathbb{R}_+ \to \mathbb{R}^{d\times d}$ is a matrix function, where we denote column i by $\beta_i(t)$, so that $\mathcal{B}(t) = (\beta_1(t), \ldots, \beta_d(t))$. When squared, the volatility parameter function $\rho(t, x)$ must be affine in x, i.e.

$$\rho(t, x)\rho(t, x)^\top = a(t) + \sum_{i=1}^{d} \alpha_i(t)x_i,$$

for matrix functions $a : \mathbb{R}_+ \to \mathbb{R}^{d\times d}$ and $\alpha_i : \mathbb{R}_+ \to \mathbb{R}^{d\times d}$. Consider now affine transformations of $\mathbf{X}$, by defining a vector function $c : \mathbb{R}_+ \to \mathbb{R}^p$ and a matrix function $\Gamma : \mathbb{R}_+ \to \mathbb{R}^{p\times d}$, thereby defining the p-dimensional process,

$$Y(t) = c(t) + \Gamma(t)X(t). \tag{2.1}$$