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On optimal periodic dividend strategies in the dual model with diffusion

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HIGHLIGHTS

- We study optimal periodic dividend strategies in the dual model with diffusion.
- Dividends are paid at random time intervals but ruin can happen at any time.
- A periodic barrier strategy is proven to be optimal in this setting.
- We study conditions under which the optimal strategy exists and is unique.
- A liquidation-at-first-opportunity strategy is optimal in some cases.

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ABSTRACT

The dual model with diffusion is appropriate for companies with continuous expenses that are offset by stochastic and irregular gains. Examples include research-based or commission-based companies. In this context, Bayraktar et al. (2013a) show that a dividend barrier strategy is optimal when dividend decisions are made continuously. In practice, however, companies that are capable of issuing dividends make dividend decisions on a periodic (rather than continuous) basis.

In this paper, we consider a periodic dividend strategy with exponential inter-dividend-decision times and continuous monitoring of solvency. Assuming hyperexponential gains, we show that a periodic barrier dividend strategy is the periodic strategy that maximizes the expected present value of dividends paid until ruin. Interestingly, a 'liquidation-at-first-opportunity' strategy is optimal in some cases where the surplus process has a positive drift. Results are illustrated.

1. Introduction

1.1. Motivation and literature review

In actuarial risk theory, the introduction of a stochastic process formulation for the surplus of a company goes back to the early 20th century. The initial criterion for assessing the stability of a company was the probability of ruin, that is, the probability that the surplus ever becomes negative; see Bühlmann (1970). However, for the specifications of the surplus models to make economic sense, their drift (expected profit) is usually assumed to be positive. In the absence of surplus leakages, surpluses will hence grow (in average) to infinity, which does not make sense. Because of this, de Finetti (1957) first introduced an alternative formulation allowing explicitly for surplus leakages, called 'dividends'. Note that by 'dividends', we consider any diminution of surplus that is made to the profit of the company's owners according to the definition of 'aggregate payout' in Allen and Michaely (2003, p. 356). Many different models for the surplus of a company with dividends have

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been explored—see Avanzi (2009) and Albrecher and Thonhauser (2009) for comprehensive reviews.

When and how much dividends should be paid (a dividend strategy) is usually determined such that the expected present value of dividends until ruin is maximized. This pure maximization of dividends paid until ruin presents some issues. One of the two mentioned by Gerber (1974) is that the resulting optimal strategies are usually not realistic. In this paper, we restrict the form of the dividend strategy in order to partially address this issue, as explained below.

In most surplus models, unrestricted optimal dividend strategies lead to very irregular dividend payments, which is arguably an unrealistic feature. In reality, companies distribute dividends at regular time intervals (for instance, quarterly or annually) on the basis of balance sheets established at similar time intervals. Albrecher et al. (2011b) were the first to study random inter-dividend-decision times (in the Cramér–Lundberg model). However, their model does not allow ruin to happen in-between dividend payment times, de facto reducing to a modified discrete time model (see also Albrecher et al., 2011a, for related optimality results). Continuous monitoring of solvency with periodic dividends were first introduced by Albrecher et al. (2011c, with constant 'intensity' to ruin when the surplus is negative) in the Brownian risk model with exponential inter-dividend-decision times and Avanzi et al. (2013, with ruin defined as the first time the surplus hits 0) in the dual model with Erlang inter-dividend-decision times.

In the dual model without diffusion, the unrestricted dividend strategy that maximizes the expected present value of dividends is a continuous barrier strategy; see Bayraktar et al. (2013), whose results are extended in Bayraktar et al. (2014), with fixed transaction costs. In the dual model with diffusion and hyperexponential gains, a similar result has been established in Avanzi et al. (2011), in presence of capital injections as well. In a regime switching Brownian risk model, the optimality of the periodic barrier strategy is studied by Wei et al. (2012) when a liquidation-at-first-opportunity is not optimal. In this paper, we generalize the results of the last two references by showing that the periodic barrier strategy is still optimal in presence of hyperexponentially distributed gains and when inter-dividend-decision times are exponential. Its associated value function has a closed form representation. We also determine when a liquidation-at-first-opportunity strategy is optimal.

1.2. Structure of the paper

Section 2 defines formally the surplus model of the dual model with diffusion, and introduces the concept of periodic dividend strategies. The optimization problem considered in this paper is set up in Section 3, where admissible and optimal periodic strategies are defined, and an appropriate verification lemma is developed and proved in conjunction with its associated Hamilton–Jacobi–Bellman equation. We construct a candidate solution to the optimization problem in Section 4. We start by determining the expected present value of dividends under an arbitrary periodic dividend barrier $b > 0$ and under a liquidation-at-first-opportunity strategy $b = 0$ in Sections 4.1 and 4.2, respectively. In Section 4.3 we show that the former takes a particular form when the optimal level $b^* > 0$, whose existence and uniqueness is discussed in Section 4.4. These candidates are proven to be indeed optimal in Section 5.

Results are illustrated in Section 6. In Section 6.1, we investigate in detail the impact of parameters on the optimal strategy. In particular, we illustrate which parameter combinations lead to a liquidation-at-first-opportunity strategy to be optimal. Next, we show the impact of dividend-decision frequencies on the periodic strategy in Section 6.2. Lastly, we compare the continuous barrier strategy with the periodic barrier strategy in Section 6.3.

2. Formulation of the surplus process

2.1. The dual model with diffusion

In the dual model with diffusion, the company surplus at time t is described as

$$U(t) = u - ct + S(t) + \sigma W(t), \quad t \geq 0, \tag{2.1}$$

where $U(0) = u \geq 0$ is the initial surplus, $c > 0$ is the expense rate per unit of time and where $\{S(t)\}$ is a compound Poisson process with intensity λ and jumps with distribution function P. The process $\{W(t)\}$ is a standard Brownian motion which is independent of $\{S(t)\}$, and which has a volatility of $\sigma > 0$ per unit of time. Throughout, we will assume that the distribution P of the jumps in $\{S(t)\}$ is hyperexponential, that is,

$$\frac{dP(y)}{dy} = p(y) = \sum_{i=1}^{n} w_i \beta_i e^{-\beta_i y}, \quad \text{for } y > 0, \tag{2.2}$$

with

$$\sum_{i=1}^{n} w_i = 1, \quad w_i > 0 \text{ for all } i,$$

$$\text{and } 0 < \beta_1 < \beta_2 < \cdots < \beta_n < \infty. \tag{2.3}$$

We define (for $t \geq 0$)

$$\mu = E[U(t+1) - U(t)] = \lambda \sum_{i=1}^{n} \frac{w_i}{\beta_i} - c \tag{2.4}$$

as the expected increment of the surplus process over one time unit (its drift). Finally, note that we assume a complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$.

The dual model is appropriate for companies with stochastic gains and deterministic expenses, such as research-based companies. In addition, the diffusion component reflects additional uncertainty in the firm's expenses and gains. All of the model assumptions are motivated in Avanzi et al. (2011, Section 1.2). For instance, (2.2) can be interpreted as n different research and development departments, each with independent exponential distributions.

2.2. Periodic dividend strategies

The surplus after distribution of dividends is defined as

$$X(t) = u - ct + S(t) + \sigma W(t) - D(t), \tag{2.5}$$

where $D(t)$ represents the aggregate dividends process (assumed to be càdlàg), with $D(0) = 0$. In a periodic dividend strategy, we assume that dividend payments can only occur at some (typically random) time points. In this paper, dividend decision times are assumed to be governed by a Poisson process $\{N_\gamma(t)\}$ with intensity γ that is also independent of $\{S(t)\}$ and $\{W(t)\}$, i.e.,

$$D(t) = \int_0^t \vartheta_s dN_\gamma(s), \tag{2.6}$$

where $\{N_\gamma(t)\}$ is $\{\mathcal{F}_t\}$-adapted and where ϑ_t is the dividend payment at time t. Dividend payouts are necessarily discrete in this setting (there cannot be continuous payments) as dividend decisions can only occur when the process $\{N_\gamma(t)\}$ has jumps. This set of dividend decision times is denoted as $\mathcal{T} = \{T_1, T_2, T_3 \ldots\}$, and the quantities $T_{k+1} - T_k$, $k \geq 0$, are the inter-dividend-decision times. In this paper, these are assumed to be exponentially distributed with mean $1/\gamma$. The dividend payout at decision time