Insurance: Mathematics and Economics 55 (2014) 261–271

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Valuing risky debt: A new model combining structural information with the reduced-form approach

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HIGHLIGHTS

- We propose a new hybrid model of credit risk.
- The dynamics of external (e.g. macroeconomic) variables is taken into account.
- An efficient closed-form approximate solution is derived.
- The model offers great flexibility to describe credit spreads.
- The model outperforms the model by Madan and Unal (2000).

ARTICLE INFO

Article history:
Received July 2013
Received in revised form December 2013
Accepted 10 February 2014

Keywords:
Default risk
Credit spread
Default intensity
Reduced-form model
Hybrid model
Structural model

ABSTRACT

A new model of credit risk is proposed in which the intensity of default is described by an additional stochastic differential equation coupled with the process of the obligor's asset value. Such an approach allows us to incorporate structural information as well as to capture the effect of external factors (e.g. macroeconomic factors) in a both parsimonious and economically consistent way. From the practical standpoint, the proposed model offers great flexibility and allows us to obtain credit spread curves of many different shapes, including double humped term structures. Furthermore, an approximate closed-form solution is derived, which is accurate, easy to implement, and allows for an efficient calibration to realized credit spreads. Numerical experiments are presented showing that the novel approach provides a very satisfactory fitting to market data and outperforms the model developed by Madan and Unal (2000).

1. Introduction

Credit risk models have become an increasingly popular field of research over the years. In fact, measuring the risk of default is crucial for many subjects which operate on the financial markets (corporate bond investors, credit derivative traders, banks, mortgage suppliers, insurance companies, etc.). In addition, credit risk models have also interesting applications in macroeconomics, where they are used, for example, to rate and analyze the debt of sovereign countries (see for example Chen et al., 2011; Gray et al., 2007; Kan, 1998).

Models of default risk are mainly developed following two common approaches: the structural approach and the reduced-form approach. According to structural models, the default event is described by means of one or more variables which are directly related to the capital structure of the firm (or, in the case of sovereign bonds, of the country) issuing the debt. Usually, it is assumed that default occurs when the value of the assets of the firm (or country) falls below a threshold level that depends on the outstanding liabilities (Bäuerle, 2002; Bernard et al., 2005; Black and Cox, 1976; Briys and de Varenne, 1997; Collin-Dufresne and Goldstein, 2001; Feng and Volkmer, 2012; Hsu et al., 2004; Longstaff and Schwartz, 1995; Merton, 1974; Zhou, 2001). These models can also incorporate other structural variables, such as, for instance, the seniority of the debt (Black and Cox, 1976), tax benefits (Andersen and Sundaresan, 1996), debt restructuring (Abýnzano et al., 2009), and liquidation costs (Andersen and Sundaresan, 1996; Leland and Toft, 1996). The structural approach has the advantage of using data and information that truly reflect the balance sheet of the obligor. Nevertheless, if the value of the total assets is modeled as a continuous-time process, default turns out to be a predictable event, and the high credit spreads that are frequently observed for short-term maturities cannot be obtained

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http://dx.doi.org/10.1016/j.insmatheco.2014.02.002
0167-6687/

(Jones et al., 1984). In order to account for high short-term spreads, some authors (see, for example Chen and Panjer, 2003; Zhou, 2001) propose structural models of default with jumps in the value of the assets. This approach, however, lacks analytical tractability (especially when default is allowed to occur at any time prior to the bond's maturity), which makes it difficult to estimate the model parameters from observed credit spreads.

Following the reduced-form approach (see for instance Duffee, 1999; Duffie and Singleton, 2003; Kijima, 2000; Kijima and Muromachi, 2000; Jarrow, 2001; Lando, 1999; Schönbucher, 2002; Liang and Wuang, 2012), the default event is modeled as the first jump of a counting process whose intensity, termed intensity of default (or default intensity), is not related to any firm-specific (or, in the case of sovereign debt, country-specific) variable, but is specified exogenously. Usually, the intensity of default is described by a stochastic differential equation in which the parameters are determined by direct calibration to market data. Reduced-form models are capable of predicting high short-term credit spreads, nevertheless they do not take into account any information about the balance sheet of the obligor.

In few words we can say that reduced-form models are successful where structural models fail, and vice versa. That is the reason why, also quite recently, some researchers have focused their efforts on developing hybrid models of default risk, i.e. models that incorporate features of both the structural and the reduced-form approach.

One of such hybrid models is proposed in Madan and Unal (1998). In particular, that work is based on the reduced-form approach, nevertheless the intensity of default, instead of being prescribed exogenously, is specified as a function of the firm's equity value. It is worth noticing that in Madan and Unal (2000) have developed another hybrid model of credit risk, in which the default intensity depends on both the value of the cash assets of the obligor and on the interest rate. This latter approach offers some advantages: complex credit spread curves can be reproduced, including double humped curves, and exact closed-form solutions can be obtained. Nevertheless, the model developed in Madan and Unal (2000) has the drawback that the credit spreads may also become negative (see Sections 4 and 5 in the present manuscript).

Another hybrid model of credit risk is proposed by Cathcart and El-Jahel in Cathcart and El-Jahel (2003, 2006). In these works it is assumed that default occurs with certainty when a "signaling variable", that is a fictitious variable incorporating all the structural information, falls below a fixed threshold level. However, default can also happen as the first jump of a counting process whose intensity is specified as a function of the interest rate in Cathcart and El-Jahel (2003) and as a function of the interest rate and of the signaling variable itself in Cathcart and El-Jahel (2006).

In summary, in Cathcart and El-Jahel (2003, 2006) and Madan and Unal (1998, 2000) the default intensity is prescribed as a *deterministic* function of some structural variable (in Cathcart and El-Jahel, 2006 and Madan and Unal, 2000 it is also a function of the interest rate). Nevertheless, the intensity of default represents the probability of *unexpected* default (default occurring in an infinitesimal time), and thus should also reflect the inter-temporal variations of external factors, such as the business cycles, the macroeconomic environment, the economic uncertainty or others. The effects of such exogenous variables on credit spreads are documented, for example, in Athanassakos and Carayannopoulos (2001), Balkan (1992), Baum and Wan (2010), Beckworth et al. (2010), Fabozzi et al. (2009), Teixeira (2007) and de Wet et al. (2009). Therefore, in this paper, in order to take into account both the complex dynamics of external variables, as well as to incorporate structural information, we propose a model of default risk where the default intensity is described by an additional stochastic differential equation coupled with the process of the firm's (or of the sovereign country's) asset value. In other words, we develop a new hybrid model in which the default intensity takes also into account the effect of external (e.g. macroeconomic) factors using a parsimonious and economically consistent dynamics.

Precisely, we assume that default occurs with certainty when the asset value, which is modeled as a geometric Brownian motion, falls below a fixed threshold level. However, the default event can also occur as the first jump of a counting process whose intensity is described by a generalized Ornstein–Uhlenbeck stochastic differential equation. In addition, in order to take into account the complex interaction between the dynamics of external variables and the obligor's capital structure, the long-run mean of the default intensity process is described by an appropriate function of the asset value.

The resulting model has the following peculiar features, which clearly appear desirable both for theoretical and economical reasons: (1) the capital structure of the obligor is properly taken into account; (2) the default intensity (i.e. the probability of unexpected default) is specified by an additional stochastic differential equation, which allows us to model complex inter-temporal dynamics of unexpected default due to macroeconomic factors; (3) the dependence of the probability of unexpected default on the asset value is incorporated.

From the practical standpoint, the approach proposed in the present manuscript offers great flexibility to describe credit spreads, and allows us to reproduce term structures of several different shapes, including double humped curves, which reflect the complex interaction between the obligor's capital structure and the dynamics of unexpected default. Note that term structures with multiple peaks are sometimes experienced in the financial markets (see Bohn, 1999; Houweling et al., 2001; Nelson and Siegel, 1987). Nevertheless, among the hybrid models of credit risk that are available in the literature, only the one in Madan and Unal (2000) is capable of reproducing this kind of curves (at least to the best of our knowledge).

It is also worth mentioning that our model is mathematically tractable, as a closed-form approximate solution can be obtained using a perturbation approach and the Laplace transform. Such an analytical solution is accurate, easy to implement, and allows for an efficient calibration to empirical data. We emphasize that an analogous approximation can be obtained also if the default intensity is modeled using a CIR-type process (in place of the Ornstein–Uhlenbeck-type process, see Appendix B).

Finally, in the present manuscript, the empirical performances of the proposed model have been tested in describing the term structure of the sovereign debt of Italy. The results obtained reveals that the novel approach provides a very satisfactory description of realized credit spreads and outperforms the model developed in Madan and Unal (2000).

2. The mathematical model

For the sake of brevity, in this section (as well as in the two sections that follow) we refer to the case of a defaultable bond issued by a firm. Nevertheless, the mathematical model that we are going to describe can also be used to price bonds issued by sovereign countries (see Section 5).

The value $V(t)$ of the firm issuing the bond is modeled as the stochastic process:

$$dV(t) = \mu V(t)dt + \sigma V(t)dW^{(1)}(t), \qquad V(t_0) = V_0, \tag{1}$$

where $V_0 > 0$, $\mu \geq 0$, $\sigma > 0$, and $W^{(1)}(t)$ is a standard Wiener process. We assume that default happens with certainty when $V(t)$ falls below a fixed threshold level $\overline{V}$ smaller than V_0.