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Arithmetic returns for investment performance measurement

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ABSTRACT

This paper introduces new money-weighted metrics for investment performance analysis, based on arithmetic means of holding period rates weighted by the investment's market values. This approach generates rates of return which measure a fund's or portfolio's performance and a fund manager's performance. It also enables to show that the Internal Rate of Return (*IRR*) is a weighted mean of holding period rates associated with interim values which differ from market values, so that value additivity is violated. The manager's Arithmetic Internal Rate of Return (*AIRR*) is shown to be the true period equivalent of the *cumulative* Time Weighted Rate of Return (*TWRR*), whereas the *period TWRR* (a geometric return) provides a different ranking. The method is easily generalized for coping with varying benchmark rates. We also cope with the practical problem of estimating interim values whenever they are not available.

1. Introduction

Insurance companies collect funds from their equityholders to sell insurance policies, whereby premiums are received. Part of the premiums is used to pay acquisition, underwriting, and administrative expenses. The remaining part is invested in financial securities, funds, financial or real estate portfolios to support the insurance writings (i.e., to assure that the company has sufficient capital to meet the future obligations). In general, investment income has a major role in the economic profitability of any firm which usually invests considerable amounts of money in financial assets. It is then vital for such companies to have reliable instruments to measure both the performance of an investment and the performance of the fund manager. While the performance of a fund manager depends only on the manager's policy of asset allocation and selection, the performance of an investment also depends on the timing and magnitude of the cash flows (deposits and withdrawals made by the company).

A common metric for measuring an investment's performance (portfolio, fund etc.) is the well-known Internal Rate of Return (*IRR*). The *IRR* is a money-weighted rate of return (*MWRR*) which explicitly accounts for the monetary amounts that flow into or out of a fund. In contrast, a rate of return which is insensitive to the amounts of money contributed or distributed is the so-called Time-Weighted Rate of Return (*TWRR*): Assuming a buy-and-hold strategy, it ignores increase or decrease in assets under management (which are the client's decisions), and so it is considered an appropriate tool for capturing managers' performances (see Dietz, 1966; Fisher, 1968; Gray and Dewar, 1971). The opposition TWRR/MWRR for measuring performance has so far characterized the literature on performance measurement (e.g. Newell, 1986; Tippett, 1994; Lerit, 1996; Geltner, 2003; Kahila, 2005; Spaulding, 2005; Le Sourd, 2007); several monographs underscore such an antagonism as well (e.g. Feibel, 2003; Bacon, 2008; Braverman, 2008; Kellison, 2009).

Building upon Magni (2013), we use an average-based approach for introducing new money-weighted metrics of performance measurement for both an investment (fund, portfolio) and a manager. The founding idea is that the rate of return of an investment depends on the capital invested; in particular, an investment's rate of return is associated with a specific invested capital and we show that, in general, it can be expressed as a mean of the investment's period rates of return, weighted by interim capital values. As a first contribution, we present a framework which enables one to reconcile in a simple way the investment's value added (i.e., wealth creation) with the investment's rate of return. This reconciliation is important, for value added is often neglected at the expense of rates of return, although it is wealth that investors aim to maximize, not rates of return.[1] We explicitly make use of the market values for deriving a fund's (portfolio's) performance, which we call

[1] Investors "can spend only dollars, not rates of return. Therefore [...] the bottom line of performance evaluation should be the analysis of value-added wealth creation, not merely an analysis of relative rates of return" (Tierney and Bailey, 1997, p. 76).

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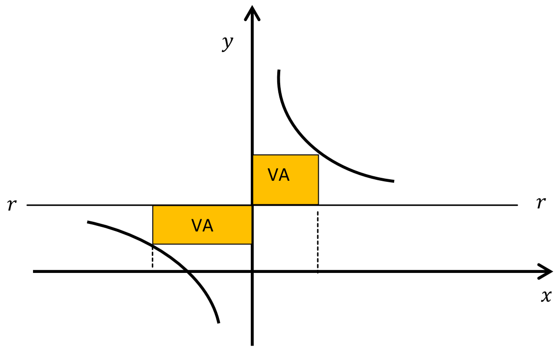

Fig. 1a. The iso-value line of a wealth-creating investment.

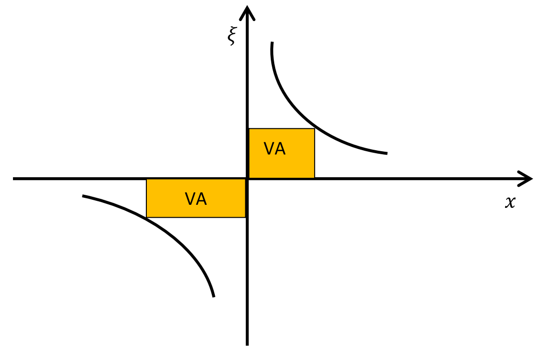

Fig. 1b. The iso-value line, net of the benchmark rate, of a wealth-creating investment.

investment's Arithmetic Internal Rate of Return (*AIRR*); we also derive a rate of return capable of measuring a manager's performance in those cases where the manager has no control over interim cash flows; we call it *manager's AIRR*. In second place, we show that the *IRR* is a rate of return on an overall invested capital which is automatically supplied by the *IRR* itself. Such an overall capital is generated by interim values which differ from the actual (i.e., market) values of the fund. We also show that the *IRR* is a weighted mean of generally time-variant period rates and that it violates value additivity. In third place, we investigate some relations between the notion of *TWRR* and the manager's *AIRR*. In particular, we distinguish *period TWRR* from *cumulative TWRR* and show that the former (which is a geometric return) is not equivalent to the latter; rather, it is the manager's *AIRR* that is the period equivalent of the cumulative *TWRR*. We then generalize the analysis allowing for benchmark rates varying over time. Finally, we show how to cope with those situations where market values are not available at each date.

The remainder of the paper is structured as follows. Section 2 summarizes the average-rate-of-return method and shows how it can be applied to a portfolio or fund where market values are employed. Section 3 shows that the *IRR* can be derived from holding period rates associated with an unbounded set of sequences of interim capital values, all of which are different from market values. Section 4 introduces the manager's *AIRR*, which captures the money manager's skills and shows that the manager's *AIRR* is the equivalent of the cumulative *TWRR*. Section 5 generalizes the approach by considering benchmark rates varying over time. Section 6 analyzes the case where market values are not available, and illustrates two procedures for overcoming the estimation problem. Some remarks conclude the paper.

2. The investment's *AIRR*

A rate of return is, by definition, "return on *capital*". In a single period, the difference between the actual investment's rate of return and the benchmark rate of return represents an excess (or active) rate of return which, multiplied by the invested capital, leads to the investment's excess return, also known as 'value added' (*VA*). A \$10 value added can be equivalently seen as an active 10% on \$100, as an active 25% on \$40, as an active 80% on \$12.5 etc. For any given *VA*, the rate of return is ambiguous if the invested capital is not specified. This implies that a rate of return depends on the capital base; all other things unvaried, if the capital base changes, the rate of return changes as well.

We generalize this simple conceptualization in order to provide a suitable framework for multiperiod investments. Let x denote capital invested and y be the rate of return corresponding to x; let *VA* be the investment's value added and r be the benchmark rate. We might tentatively express the value added as the product of the capital base, denoted as x and the excess return rate, denoted as $\xi = y - r$, just in the way described above for a one-period investment:

$$VA = x \cdot \xi = x\,(y - r)\,; \tag{2.1}$$

solving (2.1) for y, one gets the investment's rate of return as a (hyperbolic) function of the invested capital:

$$y = y\,(x) = r + \frac{VA}{x}. \tag{2.2}$$

This means that, for any given *VA*, a rate of return of an investment is associated with a specific capital. In particular, any two investments with the same value added (and same benchmark) but different invested capitals will have different rates of return. Eq. (2.2) identifies an indifference curve on the xy-plane, where different combinations of capital and return rates lead to the same value added; for this reason, we call the curve 'iso-value line' (see Fig. 1a).

The excess rate of return is

$$\xi\,(x) = y\,(x) - r \tag{2.3a}$$

and expresses the value added per unit of invested capital x:

$$\xi\,(x) = \frac{VA}{x} \Longleftrightarrow x \cdot \xi\,(x) = VA \tag{2.3b}$$

(see Fig. 1b).

In such a way, the problem of computing the financially correct rate of return boils down to selecting the correct capital base x of an investment. For a financial investment (e.g. fund, portfolio) the capital base should evidently summarize market values. Thus, x is interpretable as the overall capital, in market-value terms, invested in the investment's lifespan. Once solved the problem of selecting the appropriate capital base, a unique return rate $y(x)$ (and unique excess return rate $\xi(x)$) will be automatically derived.

Suppose a company (henceforth named "investor" or "client") deposits a monetary amount in a fund which is managed by an investment manager. The client periodically makes injections into or withdrawals from the fund. Let f_t denote the investor's cash flow at time t. Cash flows are interpreted from the point of view of the investor, so a positive cash flow is an inflow for the investor (an outflow from the fund), whereas a negative cash flow is an outflow for the investor (an inflow into the fund). The investor's cash-flow stream $\boldsymbol{f} = (f_0, f_1, \ldots, f_n)$; time n is the terminal date, when the investor liquidates the investment; obviously $f_0 < 0, f_n > 0$.[2] For each date $t = 1, 2, \ldots, n$, let b_{t-1} denote the market value of the fund at the beginning of period $[t-1, t]$ and let e_t represent the market value of the fund at the end of the same period. The

[2] From the point of view of the fund, the cash-flow stream is $-\boldsymbol{f}$.