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Fitting asset returns to skewed distributions: Are the skew-normal and skew-student good models?

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ABSTRACT

Vernic (2006), Bolancé et al. (2008), and Eling (2012) identify the skew-normal and skew-student as promising models for describing actuarial loss data. In this paper, we change the focus from the liability to the asset side and ask whether these distributions are also useful for analyzing the investment returns of insurance companies. To answer this question, we fit various parametric distributions to capital market data which has been used to describe the investment set of insurance companies. Our results show that the skew-student is an especially promising distribution for modeling asset returns such as those of stocks, bonds, money market instruments, and hedge funds. Combining the results of Vernic (2006), Bolancé et al. (2008), Eling (2012), and this paper, it appears that the skew-student is a promising actuarial tool since it describes both sides of the insurer's balance sheet reasonably well.

1. Introduction

This paper analyzes whether the skew-normal and skew-student distributions recently discussed in actuarial and finance literature are appropriate models for describing the asset returns of insurance companies. We employ frequently used datasets to describe the investments of insurance companies and fit various parametric distributions to these data. The impetus for this analysis comes from recent empirical papers that identify the skew-normal and the skew-student as useful models for analyzing actuarial loss data (Vernic, 2006; Bolancé et al., 2008; Eling, 2012). The skew-normal and the skew-student are also increasingly popular in financial modeling (De Luca et al., 2006; Adcock, 2007, 2010, 2014; Blasi and Scarlatti, 2012), especially since they are easy to interpret and easy to implement. To our knowledge, no attempts have been made in either the finance or the actuarial literature to analyze the goodness-of-fit of skewed distributions such as the skew-normal and skew-student distributions for capital market data. This paper thus contributes to both the finance and the actuarial literature.

The finance literature shows that the skew-normal and skew-student models lead to important theoretical results, especially in the field of portfolio selection and asset pricing. For example, Stein's Lemma, which is fundamental in portfolio selection, has been extended from the normal distribution to the skew-normal and the skew-student (see Adcock, 2007, 2010, 2014). It thus seems possible that the skew-normal and skew-student models might be promising tools for theoretical work in actuarial science, e.g., regarding the individual and collective risk model or in asset liability management.

In this paper we employ datasets previously used in actuarial literature to describe the asset side of insurance companies (Eling et al., 2009; Braun et al., 2013). For these capital market datasets we then analyze the goodness-of-fit of the skewed distributions compared to 12 benchmark distributions. Among the benchmark models is the classical normal distribution, which is still the most widely used approach for financial modeling, as well as several other distributions that have been used more recently to describe asset returns (e.g. the normal inverse Gaussian distribution).

Our results show that the skew-student distribution is an especially good alternative for modeling capital market returns, even when compared to alternative benchmark models discussed in recent literature. This finding extends the documented usefulness of the skewed distributions for actuarial loss data (Vernic, 2006; Bolancé et al., 2008; Eling, 2012) to the asset side. The skew-student distribution might thus be an especially promising model for actuarial modeling, both for the assets and the liabilities on an insurer's balance sheet.

The remainder of the paper is organized as follows. In Section 2, we briefly introduce the skewed distributions and benchmark models employed in the analysis. Section 3 presents the data. The parameters estimation and goodness-of-fit results are given in Section 4. In Section 5 we calculate value at risk (VaR) and tail

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value at risk (TVaR) using the estimated parameters and compare the estimation results with the empirical values. We conclude in Section 6.

2. Analyzed distributions

For the sake of brevity we only briefly describe the skewed distributions as well as the benchmark distributions analyzed in this paper. More details on skewed distributions can be found in Eling (2012) and Genton (2004); a description of the benchmark models is given in actuarial textbooks such as Mack (2002), Kaas et al. (2009), or Panjer (2007).

2.1. Skew-normal

Let $\phi(\cdot)$ be the standard normal density function, $\Phi(\cdot)$ its distribution function, and $x \in \mathbb{R}$. The probability density function (pdf) of the skew-normal distribution is then given as (see Azzalini, 1985):

$$f(x) = 2\phi(x)\,\Phi(ax). \tag{1}$$

The distribution in (1) is called skew-normal distribution with shape parameter a, i.e., $X \sim SN(0, 1, a)$. It reduces to the normal distribution for $a = 0$; if $a \to \pm\infty$, then the skew-normal becomes the half-normal distribution. Location and scale parameters can be included via the linear transformation $Y = \xi + \omega X$, which follows the skew-normal distribution $Y \sim SN(\xi, \omega^2, a)$, with $\omega > 0$ (ξ *and* ω represent the location and scale parameters).

Pourahmadi (2007) presents an alternative, intuitive representation of the skew-normal that is useful for financial modeling. $Y \sim SN(\xi, \omega^2, a)$ can be written as a weighted average of a standard normal and a half-normal variable:

$$Y = \xi + \omega X = \xi + \omega\left(\delta\,|Z_1| + \sqrt{1-\delta^2}Z_2\right), \tag{2}$$

with $\delta = a/\sqrt{1+a^2} \in [-1, 1]$. Z_1 and Z_2 are independent $N(0, 1)$ random variables. If $\delta = 0$, then Y becomes $N\left(\xi, \omega^2\right)$. Interpreting Eq. (2) in financial economics language, the return Y is driven – in addition to the location parameter ξ – by a half-Gaussian element $|Z_1|$ modulated by $\omega\delta$ and a Gaussian element Z_2 modulated by $\omega\sqrt{1-\delta^2}$. The closer the value of δ to $+1$ (-1), the more pronounced is the skewness to the right (left). Mean, variance, skewness, and kurtosis of Y then highlight the influence of the skewness parameter δ:

$$E(Y) = \xi + \omega\sqrt{2/\pi}\,\delta, \tag{3}$$

$$\mathrm{Var}(Y) = \omega^2(1 - 2\delta^2/\pi), \tag{4}$$

$$\mathrm{Skewness}(Y) = (4-\pi)/2(\delta(2/\pi)^{1/2})^3/(1-2\delta^2/\pi)^{3/2}, \tag{5}$$

$$\text{Excess Kurtosis}(Y) = 2(\pi-3)(\delta(2/\pi)^{1/2})^4/(1-2\delta^2/\pi)^2. \tag{6}$$

The mean is a linear increasing function in δ, whereas for the variance there is a quadratic link. Note that the skew-normal distribution takes values of skewness from −1 to 1. Unlike the normal distribution, it thus can be calibrated to skewed data, but the range of potential skewness values is still relatively limited.

2.2. Skew-student

The skew-student distribution allows regulating both skewness and kurtosis, which is particularly useful in modeling capital market data. The skew-normal has a kurtosis only slightly higher than the normal distribution (maximum excess kurtosis is 0.87). An appealing alternative is a skewed version of the student t distribution, introduced by Branco and Dey (2001) and further developed by Azzalini and Capitanio (2003). The standardized student t skewed distribution is defined using the transformation:

$$X = \frac{Z}{\sqrt{W/\nu}}, \tag{7}$$

with $W \sim \chi^2(\nu)$. The parameter ν represents the degrees of freedom and Z is an independent $SN(0, 1, a)$; using $N(0, 1)$ instead would produce the standard t. The linear transformation $Y = \xi + \omega X$ then has a skew-t distribution with parameters (ξ, ω, a, ν) denoted by $Y \sim ST(\xi, \omega^2, a, \nu)$. Mean and variance can be computed as follows (for the more complex expressions for skewness and kurtosis, we refer to Azzalini and Capitanio, 2003):

$$E(Y) = \xi + \omega\eta\delta, \tag{8}$$

$$\mathrm{Var}(Y) = \omega^2\left(\frac{\nu}{\nu-2} - \eta\delta^2\right), \tag{9}$$

where $\eta = \sqrt{\frac{\nu}{\pi}}\frac{\Gamma\left(\frac{1}{2}(\nu-1)\right)}{\Gamma\left(\frac{1}{2}\nu\right)}$. Eqs. (8) and (9) again highlight the influence of δ on the mean and variance of the skew-student distribution. The mean is a linear increasing function in δ, whereas the variance is a quadratic function on δ. Compared to the skew-normal distribution, the skew-student can take more extreme values for both skewness and kurtosis.

2.3. Benchmark models

The choice of benchmark models is based on their use in financial modeling. As mentioned, the most widely used distribution for modeling capital market data is the normal distribution, which is why it is used here as one of the benchmark models. Many papers, however, highlight the limitations of the normal distribution in describing capital market data and propose alternative distributions. Among the most popular of these are the normal inverse Gaussian (NIG) and other hyperbolic distributions (see, e.g., Kon, 1984; Barndorff-Nielsen, 1997; Kassberger and Kiesel, 2006).

All our benchmark models are implemented in the R packages ghyp and MASS. The skew-normal and skew-student are implemented in the R package sn. The R package ghyp contains a number of distributions popular in both finance and actuarial modeling: the normal, the student t (see Kole et al., 2007), the normal inverse Gaussian (NIG) (Barndorff-Nielsen, 1997), and the hyperbolic (Eberlein et al., 1998). In contrast to the normal distribution, some of the benchmark distributions are able to account for skewness, kurtosis, or even both (e.g., NIG, hyperbolic). Some of these distributions are related to each other, e.g., the student t, the normal inverse Gaussian, and the hyperbolic all belong to the class of generalized hyperbolic distributions. The R package MASS contains various other distributions that can be considered in a goodness-of-fit context, from which we use the Cauchy and logistic.

In the empirical part of the paper we derive maximum likelihood estimators of the best-fitting parameters and compare the benchmark distributions with the skew-normal and skew-student via the Akaike information criterion (AIC) and Kolmogorov–Smirnov goodness-of-fit tests. While the AIC might provide some basis for comparing models, it could be that none of the models are very good at describing the data. To check this, we use the Kolmogorov–Smirnov goodness-of-fit test, which analyzes whether the theoretical distributions fit the empirical data. The results of the Kolmogorov–Smirnov test can also be used to compare the skew-normal and skew-student models with the benchmark models.

Note also that a better AIC value does not necessarily mean that a model is better since there are many other aspects that actuaries need to keep in mind, such as, e.g., the risk of change of the