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Mean-chance model for portfolio selection based on uncertain measure

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ABSTRACT

This paper discusses a portfolio selection problem in which security returns are given by experts' evaluations instead of historical data. A factor method for evaluating security returns based on experts' judgment is proposed and a mean-chance model for optimal portfolio selection is developed taking transaction costs and investors' preference on diversification and investment limitations on certain securities into account. The factor method of evaluation can make good use of experts' knowledge on the effects of economic environment and the companies' unique characteristics on security returns and incorporate the contemporary relationship of security returns in the portfolio. The use of chance of portfolio return failing to reach the threshold can help investors easily tell their tolerance toward risk and thus facilitate a decision making. To solve the proposed nonlinear programming problem, a genetic algorithm is provided. To illustrate the application of the proposed method, a numerical example is also presented.

1. Introduction

Since Markowitz (1952), quantitative research on portfolio selection has attracted many scholars, and variance has been a very popular risk measurement. In his models, Markowitz proposed that expected return could be regarded as the investment return and variance the risk because the greater the variance value, the greater deviation from the expected return and thus the less likely that investors can obtain the expected return. He proposed that for a given level of variance, an optimal portfolio was obtained when the expected return was maximized; or for a given expected return, the optimal portfolio was obtained when the variance value was minimized. Since Markowitz, numerous portfolio selection models have been developed to improve and extend the mean–variance method. For example, DeMiguel and Nogales (2009) constructed the portfolio using M- and S-robust estimators instead of the classical median and mean absolute deviation to obtain the portfolio with better stability properties than the traditional minimum variance portfolio. Soleimani et al. (2009) considered minimum transaction lots, cardinality constraints and market sector capitalization as extra constraints and used a genetic algorithm (GA) to solve the problem. Anagnostopoulos and Mamanis (2010) formulated a tri-objective model to find tradeoffs between risk, return and the number of securities in the portfolio, considering quantity and class constraints. Considering that security returns are given by experts' evaluations rather than historical data in some situations, Huang (2012a) studied a new mean–variance and mean–semivariance methods for portfolio selection in these situations.

Though variance is a popular risk measurement, it is not intuitive. Since investors are better at stating their threshold levels for their goals and the maximum tolerable chances of failing to reach them than variance, scholars have showed great interest in selecting portfolios using probability of failing to reach the threshold return level or its another version, i.e., value-at-risk (VaR), as risk measurement to control risk. Some recent examples include Tsao (2010) who developed an evolutionary multi-objective approach to construct the mean–VaR efficient frontier, Durand et al. (2011) who provided an extension of evidence supporting the empirical validity and tractability of the mean–VaR efficiency concept, and Zymler et al. (2013) who developed two tractable conservative approximations for the VaR of a derivative portfolio by evaluating the worst-case VaR over all return distributions of the derivative underliers with given first- and second-order moments. Das et al. (2010), Alexander and Baptista (2011) and Baptista (2012) have also researched the way of adding probability of failing to reach the threshold return level as an extra risk measurement into Markowitz's mean–variance theory to help investors easily determine their risk-aversion coefficient and control their portfolio investment risk.

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All these researches assumed that investors have perfect information and future security returns can be fairly well reflected by the historical data. However, since security market is complex and economic environment is changing, there are situations where security returns can hardly be reflected by the historical data. In addition, nowadays many stocks are newly listed in the market. In all these situations investors lack suitable historical data. They cannot predict the security returns according to historical data but have to invite some domain experts to evaluate their belief degree toward security returns. Kahneman and Tversky (1979) have found that human beings tend to give too much weight to unlikely events. Thus, unless further suitable observed data can be obtained to revise the belief degree, subjective probability sometimes fails to model the belief degree. So far, some theories have been proposed to deal with men's belief degree toward an imprecise number such as possibility theory (Zadeh, 1978) and Dempster–Shafer theory (Dempster, 1967; Shafer, 1976). In 2007, an uncertainty theory was founded by Liu (2007) to deal with belief degree based on uncertain measure when little historical data are available. Nowadays, uncertainty theory has been used in solving problems in many optimization areas such as vehicle routing and project scheduling problems (Liu, 2010), shortest path problem (Gao, 2011), multinational project selection problem (Zhang et al., 2011), facility location problem (Gao, 2012), and inventory problem (Qin and Kar, 2013), etc. Especially, uncertainty theory was first systematically introduced into portfolio selection by Huang (2010), thus producing a theory of uncertain portfolio selection. After that, uncertain mean–semivariance model (Huang, 2012a) was discussed, uncertain risk curve (Huang, 2011) and uncertain risk index (Huang, 2012b) methods were proposed, and uncertain portfolio adjustment problem (Huang and Ying, 2013) based on risk index was studied. In this paper, we will go on exploring using uncertainty theory to develop a mean-chance method using chance of failing to reach the preset threshold level as risk measurement for portfolio selection in the situation where no suitable historical data are available and security returns are given by experts' judgments. We will first propose a factor method for evaluation of security returns based on experts' evaluations and then develop a mean-chance model. The factor method can make good use of experts' knowledge on the effects of economic environment and the companies' technological and managerial uniqueness on security returns and incorporate the contemporary relationship of security returns in the portfolio. Since it is usually easy for investors to pre-give a return threshold and the tolerance toward the chance of failing to reach this threshold level, the use of chance of failing to reach a predetermined return threshold level as risk measurement can help investors easily tell their risk tolerance level and thus facilitate an easy decision making. Since in reality there usually exist transaction costs and investors are often required by law or by their own preference to make a portfolio investment with required diversification level, and they usually prefer to make investment limitations on certain securities, we will incorporate these constraints in the model.

The rest of the paper is organized as follows. In Section 2 we will first propose a factor method for evaluation of security returns based on experts' judgments. Then we will develop a mean-chance model in Section 3 and present the deterministic equivalents of it in Section 4. Since the proposed model is a complex nonlinear programming model, we will present a GA for solving the problem in Section 5. As an illustration we will offer a numerical example in Section 6. Finally, in Section 7 we will give some concluding remarks. For better understanding of the paper, we will also briefly review some fundamentals of uncertain variables in the Appendix.

2. Factor method of evaluating security returns

Economic environment affects all security returns. In addition, the unique technology, management and other characteristics of a company, which distinguish itself from others, also affect the company's security returns independently from economic environment. Therefore, we can use two-factor model to evaluate a company's security return. One factor is economic environment factor which reflects the effect of economic environment on all security returns. It is measured in the paper by the return rate of security market index. Another factor is the company's uniqueness factor which reflects the effect of the company's unique technology, management and other characteristics on the company's security returns. Since the company's unique characteristics can result in its unique profitability from others, we compound the company's short and long term profitability to measure the company's uniqueness. Return on equity equals net income divided by shareholders' equity. It measures a company's ability to turn assets into profits. Higher values are generally favorable meaning that the company is efficient in generating income on new investment. Main business income is the income of the company's regular and main business services. Main business operating margin reflects the company's basic profitability. It tells the contribution of main business service profit to the total profit of the company and is a complementary indicator of the company's profitability. When increasing rate of return on equity and increasing rate of main business operating margin indicate the company's short term profitability, increasing rate of five-year net profit reflects the company's long term profitability. By allocating different weights to these short and long term profitability indicators, we get the uniqueness of the company. For convenience of expression, we use F to denote the factor of economic environment and f_i the ith company's uniqueness factor. In this paper we regard the two factors and security return rates as uncertain variables and propose that the ith security's return rate r_i is linearly correlated to F and f_i, i.e., $r_i = a_i + b_iF + c_if_i$. Since f_i is the factor reflecting the ith company's uniqueness, it is assumed that f_i is independent of F, and f_i and f_j are independent of each other where $i \neq j$.

Without loss of generality, we now introduce via one security the method of evaluating the security return rate $r = a + bF + cf$ based on experts' evaluations. The first step of evaluating r is to obtain the coefficient values of a, b, c, according to the experts' evaluations and the second step is to get the uncertainty distributions of F and f.

To get the coefficient values of a, b, c, first, m numbers of experts are asked to give p numbers of values that they think the factor F may take and arrange these values from small to big ones. That is, series of $F_{i/k}$ are obtained, where $i = 1, 2, \ldots, m$, $k = 1, 2, \ldots, p$, respectively, and $F_{i/1} \leq F_{i/2} \leq \cdots \leq F_{i/p}$. If for any kth value, $\max_{1\leq i\leq m} F_{i/k} - \min_{1\leq i\leq m} F_{i/k} > \varepsilon$, the domain experts are given the summary of the results and the reasons for these results, and then are asked to provide their revised estimations of the p numbers of values that the factor F may take. It is believed that during the process the opinions of the experts will converge to an appropriate answer. When $\max_{1\leq i\leq m} F_{i/k} - \min_{1\leq i\leq m} F_{i/k} \leq \varepsilon$, we calculate

$$F_k = \frac{1}{m}\sum_{i=1}^{m} F_{i/k}, \quad k = 1, 2, \ldots, p.$$

Then the data set $(F_1, F_2, \ldots, F_p)$ that the factor F may take are obtained. In a similar way, the data set $(f_1, f_2, \ldots, f_p)$ that the factor f may take can also be obtained.

Next, the experts are asked to evaluate what the security return rates will be if $F = F_k$ and $f = f_k$, $k = 1, 2, \ldots, p$. That is, from the experts the data set of

$$(y_{i/1}, F_1, f_1), (y_{i/2}, F_2, f_2), \ldots, (y_{i/p}, F_p, f_p), \quad i = 1, 2, \ldots, m$$