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Ruin measures for a compound Poisson risk model with dependence based on the Spearman copula and the exponential claim sizes

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HIGHLIGHTS

- The risk model with dependent claim amounts and interclaim times is studied.
- The dependent structure is described by Spearman copula.
- Lundeberg's generalized equation is investigated.
- We give the explicit expression of the discounted penalty function for the exponential claim size using the Laplace transform.
- The impact of the degree of dependence on the probability of ruin is studied.

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ABSTRACT

This paper is devoted to an extension to the classical compound risk model. We relax the independence assumption of claim amounts and interclaim times. The dependent structure between these random variables is described by the Spearman copula. We study the Laplace transform of the discounted penalty function and we give the explicit expression of it for the exponential claim size.

1. Introduction

The classical risk models are based on the assumption of the independence between the claim amounts and the interclaim times. Recently, some papers consider extensions to these risk models. Among them, Albrecher and Boxma (2004) investigated the generalization of the classical ruin model with the Markovian claim occurrence process. They presented the exact analytical expressions for the Laplace transform of the ruin probability. Boudreault et al. (2006) assumed that the conditional density of claim amount was a special mixture of two densities and it was dependent on the previous interclaim time. They obtained the Laplace transform of the time and probability of ruin for the large class of the claim amount distributions. Albrecher and Teugels (2006) studied the risk model with the dependent structure of the claim amounts and the interclaim times described by the copula. They obtained the exponential estimates for finite and infinite-time ruin probabilities. Cossette et al. in their papers (Cossette et al., 2008, 2010) studied the risk model with the dependent structure defined by the Farlie–Gumbel–Morgenstern copula. They derived the Laplace transform of Gerber–Shiu discounted penalty function and obtained the explicit expression for the Laplace transform of the time of ruin and the deficit at ruin for the exponential claim sizes. Chadjiconstantinidis and Vrontos (2014) assumed in this model, that the interclaim times have Erlang(n) distribution. They gave for exponential claims explicit expression for the probability of ruin and Laplace transform of time of ruin.

Ambagaspitiya (2009) relaxed the independence assumption and investigated two Sparre Andersen risk models based on the bivariate gamma distributions. He obtained the explicit expression for the probability of ruin. The model, when the joint distribution of the interclaim times and the claim amount was bivariate phase-type was studied by Badescu et al. (2009). They presented the explicit expression of the penalty function, when it depended on the deficit at ruin only. The dependent structure characterized by the conditional density function was studied by Wilmot (2007)

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and Cheung et al. (2010). Cheung et al. (2010) investigated the generalized penalty function. The authors added two variables to the Gerber–Shiu discounted penalty function: the surplus immediately after the second last claim before ruin occurs and the minimum surplus before ruin.

In this paper we investigate the compound Poisson risk process when the dependent structure of the claim amounts and the interclaim times is described by the Spearman copula and the claims have the exponential distribution. This simple copula is a convex combination of the independence and comonotonic copulas. In Section 2 we present our risk process and the general form of the discounted penalty function when the dependent structure is defined by the copula. Section 3 is devoted to the Spearman copula. The generalized Lundberg equation is provided in Section 4. We give the solution of this equation in this section. We present an explicit expression of special case of the Gerber–Shiu discounted penalty function in Section 5 using the Laplace transform. The last section is devoted to ruin probabilities. We study the impact of the degree of dependence on the probability of ruin in it.

2. Risk process

In this paper we consider the following surplus process:

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i,$$

where $u > 0$ is the initial surplus, $c > 0$ is the premium rate, $(X_i)_{i\geq 1}$ are the claim amounts and $N(t)$ is the claim number process. The $N(t)$ is the Poisson process with the interclaim times $(W_i)_{i\geq 1}$. The interclaim times $W_1, W_2, \ldots$ are the identically, exponentially distributed random variables, with the cumulative distribution function (c.d.f.) $F_W(x) = 1 - e^{-\lambda x}$ and the probability distribution function (p.d.f.) $f_W(x)$. We assume that the claims $X_1, X_2, \ldots$ are the identically, exponentially distributed random variables, too with the c.d.f. $F_X(x) = 1 - e^{-\beta x}$ and the p.d.f. $f_X(x)$. We also assume that the random vectors (X_i, W_i), $i = 1, 2, \ldots$, are independent and they are identically distributed as the canonical random vector (X, W). The components of such a vector may be dependent. The joint cumulative distribution function is denoted by $F(x, t)$.

The time of ruin T is the moment when the risk process $U(t)$ takes the negative value for the first time, i.e. $T = \inf\{t : U(t) < 0\}$. We will study the expected value of the discounted penalty function introduced by Gerber and Shiu (1998):

$$m_\delta(u) = E[e^{-\delta T} w(U(T^-), |U(T)|)\mathbf{1}_{\{T<\infty\}} | U(0) = u],$$

where $w(x, y)$, for $x, y \geq 0$, is the penalty function at the time of ruin for the surplus prior to ruin and the deficit at ruin, $\delta \geq 0$ is the force of interest and $\mathbf{1}_A$ is the indicator function of event A. When $w(x, y) = y$ we obtain the expectation of the present value of the deficit of ruin and when $w(x, y) = 1$ for every x, y, the $m_\delta(u)$ becomes the Laplace transform (LT) of the time of ruin. The infinite-time ruin probability

$$\psi(u) = P(T < \infty | U(0) = u)$$

is a special case of the Gerber–Shiu discounted penalty function $m_\delta(u)$, when $w(x, y) = 1$ and $\delta = 0$.

We know that if $c\beta \leq \lambda$, then the ruin is the certain event for any value of initial surplus u in this case, i.e. $\psi(u) = 1$. So, we assume that $c\beta > \lambda$ in our paper.

The Gerber–Shiu discounted penalty function can be given by the formula (see Cossette et al., 2010)

$$m_\delta(u) = \int_0^\infty \int_0^{u+ct} e^{-\delta t} m_\delta(u + ct - x) dF(x, t) + \int_0^\infty \int_{u+ct}^\infty e^{-\delta t} w(u + ct - x, x - u - ct) dF(x, t). \quad (1)$$

3. Dependence structure based on Spearman copula

The dependent structure of random vector (X, W) can be described by some copula $C(u_1, u_2)$. The bivariate distribution function F with marginals F_X and F_W can be written as $F(x, t) = C(F_X(x), F_W(t))$ in this case (see Nelsen, 2006). In our paper we assume, that the joint distribution of (X, W) is defined by the copula

$$C_\alpha(u_1, u_2) = (1 - \alpha)C_I(u_1, u_2) + \alpha C_M(u_1, u_2),$$

where $C_I(u_1, u_2) = u_1u_2$, $C_M(u_1, u_2) = \min(u_1, u_2)$ and $0 \leq \alpha \leq 1$. It is the family B11 in Joe (1997) and the nonnegative part of linear Spearman copula studied in Hürlimann (2004a,b). We will call it in short as the Spearman copula in our paper.

The Spearman copula is the convex combination of independent C_I and comonotonic C_M copulas. The parameter $0 \leq \alpha \leq 1$ reflects the degree of dependence. When $\alpha = 0$ we obtain the independent, classical case and for $\alpha = 1$ we have comonotonic random variables X and W. So, it is able to model a whole range of positive dependences, between the independence and comonotonicity. This simple copula lets us to investigate the influence of the degree of dependence on the value of the penalty function.

The parameter α is equal to Spearman's coefficient of correlation of the marginal random variables. This parameter of the dependence is also equal to the coefficient of upper tail dependence (see Hürlimann, 2004a,b). So, the Spearman copula leads to the simple tail dependence structure and it is a desirable property in insurance and financial modeling, mainly, in a situation where data tend to be dependent in their extreme values. The analytical evaluation of the distribution and the stop-loss transform of bivariate sums following the Spearman copula often required in such a modeling. Moreover, it is a good competitor in fitting bivariate cumulative returns (see Hürlimann, 2004a,b).

We must remember, making some mathematical transformations, that the Spearman copula has a singular component. This copula reflects the positive dependence only, but we observe such a dependence in many situations, e.g. in the earthquakes (see Boudreault et al., 2006; Nikoloulopoulos and Karlis, 2008).

The formula (1) takes the form

$$m_\delta(u) = (1 - \alpha)(I_1(u) + I_2(u)) + \alpha(I_3(u) + I_4(u)),$$

where

$$I_1(u) = \int_0^\infty \int_0^{u+ct} e^{-\delta t} m_\delta(u + ct - x) dF_I(x, t),$$

$$I_2(u) = \int_0^\infty \int_{u+ct}^\infty e^{-\delta t} w(u + ct - x, x - u - ct) dF_I(x, t),$$

$$I_3(u) = \int_0^\infty \int_0^{u+ct} e^{-\delta t} m_\delta(u + ct - x) dF_M(x, t),$$

$$I_4(u) = \int_0^\infty \int_{u+ct}^\infty e^{-\delta t} w(u + ct - x, x - u - ct) dF_M(x, t)$$

and $F_I(x, t) = C_I(F_X(x), F_W(t))$, $F_M(x, t) = C_M(F_X(x), F_W(t))$ when the dependent structure of (X, W) is defined by Spearman copula. The part $I(u) = I_1(u) + I_2(u)$ of the formula (1) connected with independence was calculated in Cossette et al. (2010). It is equal to

$$I(u) = \frac{\lambda}{c} \int_u^\infty e^{-\frac{\lambda+\delta}{c}(v-u)} \sigma(v) dv,$$

where

$$\sigma(u) = \int_0^u m_\delta(u - x) dF_X(x) + \int_u^\infty w(u, x - u) dF_X(x).$$

The copula $C_M(u_1, u_2)$ as c.d.f. focused on $\mathbf{I}^2 = [0, 1]^2$ is singular (see Nelsen, 2006). The set $D = \{(u_1, u_2) : u_1 = u_2\}$, the main diagonal of $\mathbf{I}^2$, is a support of such c.d.f.. We have $\frac{\partial^2 C_M(u_1,u_2)}{\partial u_1 \partial u_2} = 0$