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Capital allocation based on the Tail Covariance Premium Adjusted

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HIGHLIGHTS

- I define Tail Covariance Premium Adjusted and its capital allocation proportion.
- I calculate the capital allocation based Tail Covariance Premium Adjusted (TCPA).
- I illustrate the numerical applications of TCPA for the exponential cases.

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ABSTRACT

The current Solvency II process makes risk capital allocation to different business lines more and more important. This paper considers two business lines with the exponential loss distributions linked by a Farlie–Gumbel–Morgenstern (FGM) copula, modelling the dependence between them. As an allocation principle we use the Tail Covariance Premium Adjusted and obtain expressions for the allocation to the two business lines.

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1. Introduction

There is a great interest in risk measures and capital allocation because they play a central role in Solvency II. According to EIOPA (2010), Quantitative Impact Study 5 Technical Specifications, the allocation of the risk margin to the lines of business should be done according to the contribution of the lines of business to the overall Solvency Capital Requirements during the lifetime of the business.

There is a large number of capital allocation principles, which have their advantages and disadvantages, as we can see from the extensive discussion in Dhaene et al. (2011). In Bargès et al. (2009), the authors calculate the Tail Value-at-Risk (TVaR)-based allocation for risks that have exponential distributions linked by a Farlie–Gumbel–Morgenstern (FGM) copula. We will use another set of capital allocation principles for the same case.

In this paper, we will choose the capital allocation method based on the Tail Covariance Premium Adjusted (TCPA), inspired by Furman and Landsman (2006) and Overbeck (2000). The allocation principle includes the Tail Value-at-Risk, the Tail Covariance and the Tail Variance. It is a more sophisticated measure in the sense that it takes more account of heavy tails.

We use the Farlie–Gumbel–Morgenstern (FGM) Copula, like Bargès et al. (2009), to describe the dependence. One reason to use the FGM copula is its simplicity and the possibility to get closed form expression, as is illustrated in Cossette et al. (2010). Prieger (2002) suggests the FGM copula in model selection for health insurance plans. In this paper we will get a closed form expression for the Tail Covariance Premium Adjusted (TCPA) for risks with exponential loss distributions linked by a FGM copula.

The rest of the paper is organized as follows: in Section 2 the definition of the Tail Covariance Premium Adjusted (TCPA) and capital allocation method are introduced. In Section 3 we recapitulate basic properties of the exponential distributions and the FGM copula. In Section 4, we calculate the capital allocation in the case of two business lines, where both losses follow the exponential distribution with the dependence given by the FGM copula. In Section 5 the results are illustrated with numerical applications in bivariate and trivariate cases.

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2. Definition of the Tail Covariance Premium Adjusted (TCPA) based allocation

The definition of the Tail Covariance Premium Adjusted (TCPA) is based on the Tail Covariance Premium (TCovP), a risk measure introduced by Furman and Landsman (2006).

Consider risk X to be a random variable with cumulative distribution function (CDF) $F_X(x)$ and probability density function (PDF) $f_X(x)$, respectively.

The Value-at-Risk (VaR) at level q, $0 < q < 1$, of X is defined by

$$\mathrm{VaR}_q(X) = \inf\{x_q : F_X(x_q) \geq q\}. \tag{1}$$

The Tail VaR (TVaR) is defined by

$$\mathrm{TVaR}_q(X) = E[X \mid X > x_q]. \tag{2}$$

This is the expectation of loss on the right tail. We are also interested in the dispersion along the right tail. Furman and Landsman (2006) refer to this measure as Tail Variance (TV), and it is the conditional variance of the risk X, i.e.,

$$\mathrm{TV}_q(X) = \mathrm{Var}[X \mid X > x_q]. \tag{3}$$

Consider an n random variables $X_1, X_2, \ldots, X_n$, where each random variable X_i represents a risk associated with the ith business line of an insurance company or a loss from the ith asset in a portfolio of investment for an individual or an enterprise. The aggregate risk or loss is defined by the sum

$$S = \sum_{i=1}^{n} X_i.$$

Furman and Landsman (2006) defined the Tail Covariance Premium (TCovP) as

$$\mathrm{TCovP}_q(X_i \mid S) = \mathrm{TVaR}_q(X_i \mid S) + a\mathrm{TCov}_q(X_i \mid S), \tag{4}$$

where a is some non-negative constant, $i = 1, 2, \ldots, n$ and

$$\mathrm{TVaR}_q(X_i \mid S) = E[X_i \mid S > s_q] \tag{5}$$

and Tail Covariance

$$\mathrm{TCov}_q(X_i \mid S) = \mathrm{Cov}[X_i, S \mid S > s_q]. \tag{6}$$

Inspired by the allocation from Overbeck (2000),

$$E[X_i] + a\frac{\mathrm{Cov}[X_i, S]}{\sigma_S}, \tag{7}$$

we define the adjusted risk measure based on the Tail Covariance Premium (TCovP). The adjusted risk measure keeps the currency unit.

Definition 1. Tail Covariance Premium Adjusted (TCPA) is

$$\mathrm{TCPA}_q(X_i \mid S) = \mathrm{TVaR}_q(X_i \mid S) + a\frac{\mathrm{TCov}_q(X_i \mid S)}{\sqrt{\mathrm{TV}_q(S)}}, \tag{8}$$

where

$$\mathrm{TV}_q(S) = \mathrm{Var}[S \mid S > s_q]. \tag{9}$$

Furman and Landsman (2006) have proved that

$$\sum_{i=1}^{n} \mathrm{TVaR}_q(X_i \mid S) = \mathrm{TVaR}_q(S), \tag{10}$$

and

$$\sum_{i=1}^{n} \mathrm{TCov}_q(X_i \mid S) = \mathrm{TV}_q(S). \tag{11}$$

So it is straightforward to obtain

$$\sum_{i=1}^{n} \mathrm{TCPA}_q(X_i \mid S) = \mathrm{TVaR}_q(S) + a\sqrt{\mathrm{TV}_q(S)} = \mathrm{TSDP}_q(S). \tag{12}$$

The sum is exactly the Tail Standard Deviation Premium (TSDP) defined by Furman and Landsman (2006).

Denote the capital allocation proportion to the business line i by k_i, then

$$k_i = \frac{\mathrm{TCPA}_q(X_i \mid S)}{\mathrm{TSDP}_q(S)} = \frac{\mathrm{TVaR}_q(X_i \mid S) + a\frac{\mathrm{TCov}_q(X_i|S)}{\sqrt{\mathrm{TV}_q(S)}}}{\mathrm{TVaR}_q(S) + a\sqrt{\mathrm{TV}_q(S)}}. \tag{13}$$

The allocation principle satisfies the property of additivity.

3. Exponential distributions and the FGM copula

3.1. Exponential distributions

The exponential distribution is a classical distribution for random variables modelling risk. Its convenient and practical mathematical properties permit to develop explicit results.

If $X \sim \mathrm{Exp}(\lambda)$, the probability density functions (PDF) and the cumulative distribution functions (CDF) are

$$f_X(x) = \lambda e^{-\lambda x},$$
$$F_X(x) = 1 - e^{-\lambda x},$$

where $\lambda > 0$ is the rate parameter and $x \geq 0$. We know the expectation $E[X] = 1/\lambda$ and the variance $\mathrm{Var}[X] = 1/\lambda^2$.

Now we consider two business lines whose losses X_1 and X_2 follow the exponential distribution: $X_i \sim \mathrm{Exp}(\lambda_i)$ for $i = 1, 2$. To simplify the calculation, we assume that $\lambda_1 \neq \lambda_2, \lambda_1 \neq 2\lambda_2$ or $2\lambda_1 \neq \lambda_2$. The adjusted results can be obtained without these constraints.

3.2. FGM copula

A dependence structure for (X_1, X_2) based on the FGM copula is introduced.

3.2.1. Definition

Theorem 1 (*Sklar's Theorem*)**.** *For any bivariate distribution function $H(x, y)$, let $F(x) = H(x, \infty)$ and $G(y) = H(\infty, y)$ be the univariate marginal probability distribution functions. Then there exists a copula C such that*

$$H(x, y) = C(F(x), G(y)).$$

The copulas of the Farlie–Gumbel–Morgenstern family are defined by

$$C_\theta(u_1, u_2) = u_1 u_2(1 + \theta(1 - u_1)(1 - u_2))$$

for $u_i \in [0, 1]$, $i = 1, 2$, and dependence parameter $\theta \in [-1, 1]$. We simulated 500 observations from the two extreme members ($\theta = -1$ and $\theta = 1$) of this family using the R package copula, see Fig. 1.

3.2.2. Dependence

Definition 2. Let (X_1, Y_1) and (X_2, Y_2) be independent and identically distributed random vectors. Then the population version of Kendall's tau is defined as:

$$\tau = \tau_{X,Y} = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0].$$

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