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On dividend strategies with non-exponential discounting

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HIGHLIGHTS

- The dividend maximization problem is studied with a non-constant discount rate.
- An equilibrium HJB-equation is considered for this time-inconsistent control problem.
- Equilibrium strategies and value functions are obtained in two examples.

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ABSTRACT

In this paper, we study the dividend maximization problem with a non-constant discount rate in a diffusion risk model. We assume that the dividends can only be paid at a bounded rate and restrict ourselves to Markov strategies. This is a time inconsistent control problem. The equilibrium HJB-equation is given and the verification theorem is proven for a general discount function. Considering a mixture of exponential discount functions and a pseudo-exponential discount function, we get equilibrium dividend strategies and the corresponding equilibrium value functions by solving the equilibrium HJB-equations.

1. Introduction

Since it was proposed by De Finetti (1957), the optimization of dividend payments has been investigated by many researchers under various risk models. This problem is usually phrased as the management's problem of determining optimal timing and size of dividend payments in the presence of bankruptcy risk. For more literature on this problem, we refer the reader to a recent survey paper by Avanzi (2009).

In the very rich literature, a common assumption is that the discount rate is constant over time so the discount function is exponential. However, some empirical studies of human behavior suggest that the assumption of constant discount rate is unrealistic, see, e.g., Thaler (1981), Ainslie (1992) and Loewenstein and Prelec (1992). Indeed, there is experimental evidence that people are impatient about choices in the short term but are more patient when choosing between long-term alternatives. More precisely, events in the near future tend to be discounted at a higher rate than events that occur in the long run. Considering such an effect, individual behavior is best described by the hyperbolic discounting (see Phelps and Pollak, 1968), which has been extensively studied in the areas of microeconomics, macroeconomics, and behavioral finance, such as Laibson (1997) and Barro (1999) among others.

However, difficulties arise when we try to solve an optimal control problem with a non-constant discount rate by the standard dynamic programming approach. In fact, the standard optimal control techniques give rise to time inconsistent strategies, i.e., a strategy that is optimal for the initial time may be not optimal later. This is the so-called time inconsistent control problem and the classical dynamic programming principle does no longer hold. Strotz (1955) studies the time inconsistent problem within a game theoretic framework by using Nash equilibrium points. They seek the equilibrium policy as the solution of a subgame-perfect equilibrium where player t can be viewed as the future incarnation of the decision-maker at time t.

Recently, there is an increasing attention in the time inconsistent control problem due to the practical applications in economics and finance. A modified HJB equation is derived in Marín-Solano

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and Navas (2010) which solves an optimal consumption and investment problem with the non-constant discount rate for both naive and sophisticated agents. A similar problem is also considered by another approach in Ekeland and Lazrak (2006) and Ekeland and Pirvu (2008), who provide the precise definition of the equilibrium concept in continuous time for the first time. They characterize the equilibrium policies through the solutions of a flow of a BSDE, and they show, for a special form of the discount factor, that this BSDE reduces to a system of two ODEs which has a solution. Considering the hyperbolic discounting, Ekeland et al. (2012) study the portfolio management problem for an investor who is allowed to consume and take out life insurance, and they characterize the equilibrium strategy by an integral equation. Following this definition of the equilibrium strategy, Björk and Murgoci (2010) studied the time-inconsistent control problem in a general Markov framework, and derived the equilibrium HJB-equation together with the verification theorem. Björk et al. (2014) studied Markowitz's optimal portfolio problem with state-dependent risk aversion by utilizing the equilibrium HJB-equation obtained in Björk and Murgoci (2010).

In this paper, we revisit the dividend maximization problem with a general discount function in a diffusion risk model. We assume that the dividends can only be paid at a bounded rate and restrict ourselves to Markov strategies. We use the equilibrium HJB-equation to solve this problem. In contrast to the papers mentioned above which consider a fixed time horizon or an infinite time horizon, in the dividend problem the ruin risk should be taken into account and the time horizon is a random variable (the time of ruin). Thus, the equilibrium HJB-equation given in this paper looks different to the one obtained in Björk and Murgoci (2010). We first give the equilibrium HJB-equation which is motivated by Yong (2012) and the verification theorem for a general discount function. Then we solve the equilibrium HJB-equation for two special non-exponential discount functions: a mixture of exponential discount functions and a pseudo-exponential discount function. For more details about these discount functions, we refer the reader to Ekeland and Lazrak (2006) and Ekeland and Pirvu (2008). Under the mixture of exponential discount functions, our results show that if the bound of the dividend rate is small enough, then the equilibrium strategy is to always pay the maximal dividend rate; otherwise, the equilibrium strategy is to pay the maximal dividend rate when the surplus is above a barrier and pay nothing when the surplus is below the barrier. Given some conditions, the results are similar under the pseudo-exponential discount function. These features of the equilibrium dividend strategies are similar to the optimal strategies obtained in Asmussen and Taksar (1997) who consider the exponential discounting in the diffusion risk model.

The remainder of this paper is organized as follows. The dividend problem and the definition of an equilibrium strategy are given in Section 2. The equilibrium HJB-equation and a verification theorem are presented in Section 3. In Section 4, we study two cases with a mixture of exponential discount functions and a pseudo-exponential discount function.

2. The model

In the case of no control, the surplus process is assumed to follow

$$dX_t = \mu dt + \sigma dW_t, \quad t \geq 0,$$

where μ, σ are positive constants and $\{W_t\}_{t\geq 0}$ is a one-dimensional standard Brownian motion on a filtered probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathsf{P}\right)$ satisfying the usual hypotheses. The filtration $\{\mathcal{F}_t\}_{t\geq 0}$ is completed and generated by $\{W_t\}_{t\geq 0}$.

A dividend strategy is described by a stochastic process $\{l_t\}_{t\geq 0}$. Here, $l_t \geq 0$ is the rate of dividend payout at time t which is assumed to be bounded by a constant $M > 0$. We restrict ourselves to the feedback control strategies (Markov strategies), i.e. at time t, the control l_t is given by

$$l_t = \pi(t, x),$$

where x is the surplus level at time t and the control law $\pi : [0,\infty) \times [0,\infty) \to [0, M]$ is a Borel measurable function. In Section 4, we need to distinguish the cases with $M < \mu$ and $M \geq \mu$, and to be more careful with the former case when we verify the strategy conjectured is indeed an equilibrium strategy (see Corollaries 4.3 and 4.8).

When applying the control law π, we denote by $\{X_t^\pi\}_{t\geq 0}$ the controlled risk process. Considering the controlled system starting from the initial time $t \in [0,\infty)$, $\{X_s^\pi\}$ evolves according to

$$\begin{cases} dX_s^\pi = \mu ds + \sigma dW_s - \pi(s, X_s^\pi)ds, \quad s \geq t, \\ X_t^\pi = x. \end{cases} \tag{2.1}$$

Let

$$\tau_t^\pi := \inf\left\{s \geq t : X_s^\pi \leq 0\right\}$$

be the time of ruin under the control law π. Without loss of generality, we assume that $X_s^\pi \equiv 0$ for $s \geq \tau_t^\pi$.

Let $h : [0,\infty) \to [0,\infty)$ be a discount function which satisfies $h(0) = 1$, $h(t) \geq 0$ and $\int_0^\infty h(t)dt < \infty$. Furthermore, h is assumed to be continuously differentiable on $[0,\infty)$ and $h'(t) \leq 0$.

Definition 2.1. A control law π is said to be admissible if it satisfies: $0 \leq \pi(t,x) \leq M$ for all $(t,x) \in [0,\infty) \times [0,\infty)$, $\pi(t,0) \equiv 0$ for all $t \in [0,\infty)$. We denote by Π the set of all admissible control laws.

For a given admissible control law π and an initial state $(t,x) \in [0,\infty) \times [0,\infty)$, we define the return function V^π by

$$V^\pi(t,x) = \mathsf{E}_{t,x}\left[\int_t^{\tau_t^\pi} h(z-t)\pi(z, X_z^\pi)dz\right],$$

where $\mathsf{E}_{t,x}[\cdot]$ is the expectation conditioned on the event $\{X_t^\pi = x\}$. Note that for any admissible strategy $\pi \in \Pi$, we have

$$\mathsf{E}_{t,x}\left[\int_t^{\tau_t^\pi} \left|h(z-t)\pi(z, X_z^\pi)\right| dz\right]$$
$$\leq M\int_0^\infty h(t)dt < \infty, \quad \forall (t,x) \in [0,\infty)\times[0,\infty), \tag{2.2}$$

which means the performance functions $V^\pi(t,x)$ are well-defined for all admissible strategies.

In classical risk theory, the optimal dividend strategy, denoted by π^*, is an admissible strategy such that

$$V^{\pi^*}(t,x) = \sup_{\pi\in\Pi} V^\pi(t,x).$$

However, in our settings, this optimization problem is time-inconsistent in the sense that the Bellman optimality principle fails.

Similar to Ekeland and Pirvu (2008) and Björk and Murgoci (2010), we view the entire problem as a non-cooperative game and look for Nash equilibria for the game. More specifically, we consider a game with one player for each time t, where player t can be regarded as the future incarnation of the decision maker at time t. Given state (t,x), player t will choose a control action $\pi(t,x)$, and she/he wants to maximize the functional $V^\pi(t,x)$. In the continuous-time model, Ekeland and Lazrak (2006) and Ekeland and Pirvu (2008) give the precise definition of this equilibrium strategy for the first time. Intuitively, equilibrium strategies are the strategies such that, given that they will be implemented in the future, it is optimal to implement them right now.