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Individual loss reserving using paid–incurred data

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ABSTRACT

This paper develops a stochastic model for individual claims reserving using observed data on claim payments as well as incurred losses. We extend the approach of Pigeon et al. (2013), designed for payments only, towards the inclusion of incurred losses. We call the new technique the *individual Paid and Incurred Chain* (iPIC) reserving method. Analytic expressions are derived for the expected ultimate losses, given observed development patterns. The usefulness of this new model is illustrated with a portfolio of general liability insurance policies. For the case study developed in this paper, detailed comparisons with existing approaches reveal that iPIC method performs well and produces more accurate predictions.

1. Introduction

An insurance company is liable for the claims generated by the contracts sold to clients. The insurer will hold capital aside to meet future liabilities attached to incurred claims. He thus must predict, with maximum accuracy, the total amount necessary to pay claims that he has legally committed to cover for. This is the job of a reserving actuary. Our paper presents a novel framework for reserving using individual claim data, combining two sources of information, namely *claim payments* and *incurred losses*. On one hand, we extend the framework developed in Pigeon et al. (2013) for reserving with claim payments registered for individual claims, and now enable consistent handling of paid and incurred information. On the other hand, we extend remarkable strategies for claims reserving with paid–incurred information summarized in run-off triangles (see Quarg and Mack, 2004, Posthuma et al., 2008, Wüthrich and Merz, 2010 and Happ and Wüthrich, 2013) to the setting of individual claims.

Fig. 1 illustrates the run-off (or development) process of a non-life insurance claim. A claim occurs at a certain point in time (t_1), consequently it is declared to the insurer (at t_2), possibly after a period of delay, and one or several *claim payments* follow (at time points t_3, t_4, t_5) until the settlement (i.e. closure, at time t_6) of the claim. Insurance companies distinguish *Reported But Not Settled* (RBNS) claims and *Incurred But Not Reported* (IBNR) claims. For an RBNS claim occurrence and declaration take place before the present moment (say τ) and the settlement occurs afterwards (i.e. $\tau \geqslant t_2$ and $\tau < t_6$ in Fig. 1). An IBNR claim has occurred before the present moment, but its declaration and settlement follow afterwards (i.e. $t_1 \leq \tau < t_2$ in Fig. 1). The majority of techniques for loss reserving (see Wüthrich and Merz, 2008) aggregate information on the development of individual claims into run-off triangles. Fig. 1 visualizes this operation and Fig. 2 is an example of a run-off triangle.

Reserving actuaries typically not only consider *claim payments* (the upper part in Fig. 1) when evaluating reserves. They also analyze *incurred losses* (the lower part in Fig. 1), especially in lines of business with large settlement delays and in reinsurance. Incurred losses are the sum of cumulative claim payments and *case estimates*. Such case estimates are set by an experienced case handler and express the expert's current estimate of the outstanding loss on an individual claim. Thus, case estimates might be adjusted or corrected throughout the development of a claim. Incurred losses therefore evolve, as Fig. 1 illustrates. The incurred loss does not change when the case estimate is automatically adjusted by subtracting the paid amount (at the date of payment). Indeed, changes in claim payments and case estimates then annihilate and the incurred loss is unchanged. Incurred loss adjustments and claim payments should not necessarily occur at the same time. Typically, the initial case estimate (and thus: incurred loss) is determined by experts right after reporting of the claim. These features are illustrated in Fig. 1.

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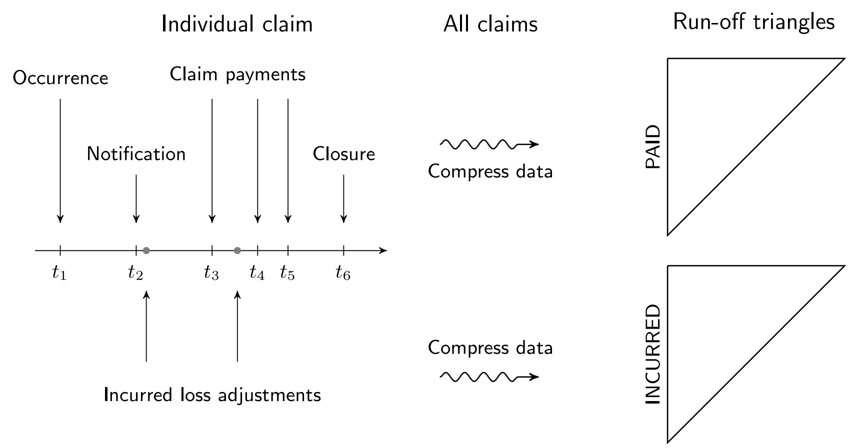

Fig. 1. Time line representing the development of a non-life claim. The upper part illustrates claim payments over time, and the lower part shows incurred loss adjustments throughout the development of a claim. This information is then summarized in two run-off triangles: one for claim payments and one for incurred losses.

	1	2	3	4	5	6	7	8	9
1997	271148	905934	1284478	1861430	2473545	2631995	2786279	3196535	3210059
1998	213628	712468	1048113	1424123	1735178	1802001	2018831	2114466	NA
1999	255561	870035	1325213	1640168	1943254	2290108	2452256	NA	NA
2000	260538	905787	1421300	2016922	2508790	2966609	NA	NA	NA
2001	458840	1418720	2077303	2754281	3302786	NA	NA	NA	NA
2002	318890	1015278	1663586	2212191	NA	NA	NA	NA	NA
2003	298100	1216496	1964751	NA	NA	NA	NA	NA	NA
2004	328135	1371792	NA	NA	NA	NA	NA	NA	NA
2005	554960	NA	NA	NA	NA	NA	NA	NA	NA

Fig. 2. Bodily Injury: cumulative triangle with claim payments, aggregated by occurrence and development year.

In current actuarial practice both sources of information are summarized in separate triangles, see Fig. 1 or Table 1 in Wüthrich and Merz (2010). The loss reserving actuary can either work with a single channel of information (e.g. only claim payments), or can use a paid–incurred method designed for triangles. Section 1 in Wüthrich and Merz (2010) is an excellent overview of the milestones in the literature covering such methods.

Recently, the necessity and appropriateness of the use of run-off triangles has been challenged. Several authors have proposed extensions of traditional loss reserving techniques, based on a single run-off triangle, towards the use of additional data sources. For example, the Double Chain-Ladder (DCL) method (see Verrall et al., 2010, Martinez et al., 2011 and Martinez et al., 2012) extends the traditional chain-ladder framework towards the use of two run-off triangles, with numbers of reported claims on one hand and claim payments on the other hand. Martinez et al. (2013) extend the DCL by using information on incurred losses in a Bornhuetter–Ferguson way (see Bornhuetter and Ferguson, 1972). Building upon the fundamental work of Norberg (1993), Haastrup and Arjas (1996) and Norberg (1999), Antonio and Plat (accepted for publication) develop a true micro-level approach which models the development of individual claims in continuous time. Antonio and Plat (accepted for publication) design their method for payments only, though information on incurred losses is incorporated by using the initial case estimate as a covariate when modeling severities. Drieskens et al. (2012), Rosenlund (2012) and Pigeon et al. (2013) work in discrete time and aggregate payments per time period (e.g. a development year), but keep the claim specific time line. Taking the perspective of a reinsurer, Drieskens et al. (2012) develop a model for individual claims reserving using (exclusively) information on incurred losses.

The ambition of this paper is the design of a stochastic method for individual claims reserving, which enables a consistent combination of information on claim payments and incurred losses. We develop such a method by extending the discrete time approach in Pigeon et al. (2013) towards the use of incurred losses. The relevance attached by practicing actuaries to incurred loss information motivates our extension. We call the new technique the *individual Paid and Incurred Chain* (iPIC) reserving method.

The paper is organized as follows. We introduce the statistical model in Section 2 and we present our theoretical results in Section 3. We devote Section 4 to the parameter estimation problem and we perform a case study in Section 5. Finally, we conclude in Section 6. Some technical developments are gathered in an Appendix, for the sake of completeness.

2. Model specification and assumptions

Our starting point is a data set with detailed information on the development of claim payments and incurred losses of individual claims. More specifically, the model uses the occurrence date, the reporting date, the date(s) and size(s) of claim payment(s) and incurred loss adjustment(s), and the closure date of the claim. We illustrate the available information in continuous time in the first three columns of Table 1. In column 1, '*P*' refers to cumulative payments, and in column 2 '*I*' is for incurred loss, as recorded on a MM/DD/YY basis. In the example in Table 1 the claim occurs on April 26, 1997. It is reported to the insurance company on November 28, 1997. The initial case estimate (and thus incurred loss) is set on December 11, 1997 for an amount of 4402 euro. Payments and incurred loss adjustments follow as indicated in the table. Finally, the claim closes on December 3, 2003.

2.1. Notation

We leave the continuous time framework from Fig. 1 and work in discrete time (e.g. with periods of one year). We denote the kth claim in the data base with k and the number of claims from occurrence period i by K_i, $i = 1, \ldots, I$, where I is the number of occurrence periods considered. We work with occurrence periods expressed in years. Our discrete time framework has the same set up as in Pigeon et al. (2013), but is now extended to an additional