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Asymptotic finite-time ruin probability for a bidimensional renewal risk model with constant interest force and dependent subexponential claims

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ABSTRACT

This paper considers a bidimensional renewal risk model with constant interest force and dependent subexponential claims. Under the assumption that the claim size vectors form a sequence of independent and identically distributed random vectors following a common bivariate Farlie–Gumbel–Morgenstern distribution, we derive for the finite-time ruin probability an explicit asymptotic formula.

1. Introduction

In this paper, we consider a bidimensional renewal risk model, in which an insurer simultaneously operates two kinds of business sharing a common claim-number process. Concretely speaking, the bidimensional surplus process of the insurer is described as

$$\begin{pmatrix} U_{1r}(t) \\ U_{2r}(t) \end{pmatrix} = \begin{pmatrix} xe^{rt} \\ ye^{rt} \end{pmatrix} + \begin{pmatrix} \int_{0-}^{t} e^{r(t-s)} C_1(\mathrm{d}s) \\ \int_{0-}^{t} e^{r(t-s)} C_2(\mathrm{d}s) \end{pmatrix} - \begin{pmatrix} \sum_{i=1}^{N(t)} X_i e^{r(t-\tau_i)} \\ \sum_{i=1}^{N(t)} Y_i e^{r(t-\tau_i)} \end{pmatrix}, \quad t \geq 0, \tag{1.1}$$

where $r \geq 0$ denotes the constant interest force, (x, y) the initial surplus vector, $(C_1(t), C_2(t))$ the vector of the total premium accumulated up to time t with the nondecreasing and right continuous components satisfying $(C_1(0), C_2(0)) = (0, 0)$, and $\{(X_i, Y_i);\ i \geq 1\}$ the sequence of claim size vectors whose common arrival times $\tau_1, \tau_2, \ldots$ constitute a renewal claim-number process $\{N(t); t \geq 0\}$ with finite renewal function $\lambda(t) = \mathbb{E}N(t) = \sum_{i=1}^{\infty} \mathbb{P}(\tau_i \leq t)$.

Throughout this paper, $\{(X_i, Y_i); i \geq 1\}$ is assumed to be a sequence of independent and identically distributed (i.i.d.) random vectors with generic vector (X, Y) whose marginal distribution functions are $F = 1 - \overline{F}$ on $[0, \infty)$ and G on $[0, \infty)$, respectively. We further assume that $\{(X_i, Y_i);\ i \geq 1\}$, $\{(C_1(t), C_2(t)); t \geq 0\}$ and $\{N(t); t \geq 0\}$ are mutually independent.

Define the finite-time ruin probability of risk model (1.1) as

$$\psi(x, y; T) = \mathbb{P}(T_{\max} \leq T \,|\, (U_{1r}(0), U_{2r}(0)) = (x, y)), \quad T > 0,$$

where

$$T_{\max} = \inf\{t > 0 : \max\{U_{1r}(t), U_{2r}(t)\} < 0\}$$

denotes the ruin time with $\inf \emptyset = \infty$ by convention. We aim to seek the precise asymptotic expansion for $\psi(x, y; T)$ as $(x, y) \to (\infty, \infty)$ with fixed T.

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The asymptotic behavior of finite-time ruin probabilities for risk model (1.1) and its variants (e.g., with $r = 0$, with constant premium rate, with Poisson claim-number process, or with Brownian perturbation) has been widely investigated in recent years. See Li et al. (2007), Liu et al. (2007), Chen et al. (2011), Zhang and Wang (2012), Chen et al. (2013a,b), Hu and Jiang (2013), and Yin et al. (2013), among others.

All of the aforementioned literatures assumed that the claim size vector (X, Y) consists of the independent components, i.e., $\{X_i;\ i \geq 1\}$ and $\{Y_i;\ i \geq 1\}$ are independent. However, such a complete independence assumption was proposed mainly for the mathematical tractability rather than the practical relevance. Therefore, to improve this defect of the traditional bidimensional risk models, we assume in the present paper that (X, Y) follows a bivariate Farlie–Gumbel–Morgenstern (FGM) distribution. Recall that a bivariate FGM distribution with marginal distribution functions F and G is given as

$$\Pi(x, y) = F(x)G(y)\left(1 + \theta\overline{F}(x)\overline{G}(y)\right), \quad \theta \in [-1, 1]. \tag{1.2}$$

Trivially, if $\theta = 0$ then (1.2) reduces to a joint distribution function of two independent random variables.

Additionally, instead of restricting the claim size distributions to some proper subclasses of the subexponential class or considering the special Poisson claim-number process as done in the existing literatures mentioned above, we greatly weaken the technical assumptions in our framework and conduct the analysis under the general renewal risk model with subexponential claims.

In the rest of this paper, Section 2 presents our main result after introducing necessary preliminaries and Section 3 proves the main result after preparing some useful lemmas. Finally, in the Appendix we show a routine (but a bit tedious) proof of Lemma 3.2, which is a crucial result for our purpose.

2. Preliminaries and main result

A distribution function V on $[0, \infty)$ is said to belong to the subexponential class, written as $V \in \mathscr{S}$, if $\overline{V}(x) > 0$ for all $x \geq 0$ and the relation

$$\lim_{x\to\infty} \frac{\overline{V^{n*}}(x)}{\overline{V}(x)} = n$$

holds for all (or, equivalently, for some) $n \geq 2$, where V^{n*} is the n-fold convolution of V with itself. It is known that if $V \in \mathscr{S}$ then $V \in \mathscr{L}$, which stands for the class of long-tailed distributions characterized by $\overline{V}(x) > 0$ for all $x \geq 0$ and the relation

$$\lim_{x\to\infty} \frac{\overline{V}(x+z)}{\overline{V}(x)} = 1, \quad z \in (-\infty, \infty).$$

One of the most important subclasses of $\mathscr{S}$ is the class of distributions with regularly-varying tails. By definition, a distribution function V is said to belong to the class of distributions with regularly-varying tails if $\overline{V}(x) > 0$ for all $x \geq 0$ and the relation

$$\lim_{x\to\infty} \frac{\overline{V}(xz)}{\overline{V}(x)} = z^{-\alpha}, \quad z > 0 \tag{2.1}$$

holds for some $\alpha \geq 0$. In this case, we write $V \in \mathscr{R}_{-\alpha}$. By Theorem 1.5.2 of Bingham et al. (1987), relation (2.1) holds uniformly on all compact z-sets of $(0, \infty)$.

Hereafter, all limit relations hold as $(x, y) \to (\infty, \infty)$ unless otherwise stated. As usual, for two positive bivariate functions $f(\cdot, \cdot)$ and $g(\cdot, \cdot)$, we write $f \lesssim g$ or $g \gtrsim f$ if $\limsup f/g \leq 1$ and write $f \sim g$ if both $f \lesssim g$ and $f \gtrsim g$. To avoid triviality, a nonnegative random variable is always assumed to be nondegenerate at 0.

Now, we are ready to present our main result of this paper.

Theorem 2.1. *Consider risk model (1.1) in which $\{(X, Y), (X_i, Y_i);\ i \geq 1\}$ is a sequence of i.i.d. random vectors following a common bivariate FGM distribution (1.2) with $\theta \in (-1, 1]$. Let the distribution functions of X and Y be $F \in \mathscr{S}$ and $G \in \mathscr{S}$, respectively. Let $T > 0$ such that $\lambda(T) > 0$.*

(i) *If $\theta \in (-1, 0]$ then*

$$\begin{aligned}\psi(x, y; T) &\sim \iint\limits_{\substack{s,t\geq 0\\ s+t\leq T}} \left[\overline{F}\left(xe^{r(t+s)}\right)\overline{G}\left(ye^{rt}\right) + \overline{F}\left(xe^{rt}\right)\overline{G}\left(ye^{r(t+s)}\right)\right]\lambda(ds)\,\lambda(dt)\\ &\quad + (1+\theta)\int_{0-}^{T} \overline{F}\left(xe^{rt}\right)\overline{G}\left(ye^{rt}\right)\lambda(dt).\end{aligned} \tag{2.2}$$

(ii) *If $\theta \in (0, 1]$ and $\mathbb{E}\rho^{N(T)} < \infty$ for some $\rho > 1 + \theta$, then relation (2.2) holds.*

Remark 2.1. In Theorem 2.1, we have to exclude the case of $\theta = -1$, for which our current treatment fails to give a precise asymptotic formula; see, e.g., relations (3.5), (3.10), (3.14), and Remark A.1 below. Actually, in a recent contribution, Chen studied a discrete-time unidimensional risk model with the FGM structure and also excluded such a critical case in the main results due to the similar reason; see Theorems 3.1 and 3.2 of Chen (2011).

Remark 2.2. Since the moment generating function of $N(T)$ is analytic in a neighborhood of 0 (see, e.g., Stein, 1946), there must be some $\rho > 1$ such that $\mathbb{E}\rho^{N(T)} < \infty$ and, hence, the conditions of Theorem 2.1(i) and (ii) can be merged as $\mathbb{E}\rho^{N(T)} < \infty$ for some $\rho > 1 + \theta^+$ with $\theta^+ = \max\{\theta, 0\}$.

Moreover, by the uniformity of relation (2.1) mentioned before, if $F \in \mathscr{R}_{-\alpha}$ and $G \in \mathscr{R}_{-\alpha}$ for some $\alpha \geq 0$ and $\{N(t); t \geq 0\}$ is a Poisson process with intensity $\lambda > 0$ (implying $\lambda(dt) = \lambda \cdot dt$ and $\mathbb{E}\rho^{N(T)} < \infty$ for all $\rho > 0$ and $T > 0$), then the right-hand side of (2.2) can be further expanded to a much more transparent form. Therefore, we have the following corollary immediately.

Corollary 2.1. *Consider risk model (1.1) in which $\{(X, Y), (X_i, Y_i);\ i \geq 1\}$ is a sequence of i.i.d. random vectors following a common bivariate FGM distribution (1.2) with $\theta \in (-1, 1]$. Let the distribution functions of X and Y be $F \in \mathscr{R}_{-\alpha}$ and $G \in \mathscr{R}_{-\alpha}$ for some $\alpha \geq 0$, respectively, and let $\{N(t); t \geq 0\}$ be a Poisson process with intensity $\lambda > 0$. Then, it holds for any $T > 0$ that*

$$\psi(x, y; T) \sim \begin{cases} \left[\dfrac{\lambda^2}{\alpha^2 r^2}\left(1 - e^{-\alpha r T}\right)^2 + \dfrac{\lambda(1+\theta)}{2\alpha r}\left(1 - e^{-2\alpha r T}\right)\right]\overline{F}(x)\overline{G}(y), & \alpha, r > 0,\\ \left[\lambda^2 T^2 + \lambda T(1+\theta)\right]\overline{F}(x)\overline{G}(y), & \alpha = 0 \text{ or } r = 0.\end{cases}$$

3. Proof of Theorem 2.1

3.1. Lemmas

The first lemma below is a restatement of Lemma 3.1 of Hao and Tang (2008).

Lemma 3.1. *Let $Z_1, Z_2, \ldots$ be a sequence of independent random variables with distribution functions $V_1, V_2, \ldots$, respectively. Assume that there is a distribution function $V \in \mathscr{S}$ such that $\overline{V}_i(x) \sim l_i\overline{V}(x)$ with some positive constant l_i for each $i \geq 1$. Then, for each $n \geq 1$ and any $0 < a \leq b < \infty$, it holds uniformly for $(c_1, \ldots, c_n) \in [a, b]^n$ that*

$$\mathbb{P}\left(\sum_{i=1}^{n} c_i Z_i > x\right) \sim \sum_{i=1}^{n} \overline{V}_i(x/c_i), \quad x \to \infty.$$