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Pricing inflation-linked variable annuities under stochastic interest rates

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ABSTRACT

Equities have long been dubbed the natural hedge against inflation. However, empirical findings have implied just the opposite, that there exists a negative correlation between stock returns and inflation. The rising inflation and slowing economic growth that we are experiencing in today's market environment pose an even greater threat to the general investors, especially on their retirement planning. In this paper, we present various inflation-linked variable annuities which are designed to help investors protect their portfolios from inflation risk. Assuming a Gaussian HJM framework for the nominal and real term structures, closed-form pricing formulas are obtained for these inflation-linked annuity products.

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1. Introduction

One of the most important objectives of retirement planning is to ensure that future consumption will not fall below a minimum acceptable standard of living. However, Bodie (2003) and a recent report by the World Bank (Rocha et al., 2011) point out that the investment and traditional annuity products in most of the retirement portfolios have the glaring defect that they are not protected against inflation. In the US, this is due in no small part to the limited offering of inflation-linked annuities (Brown et al., 2002). Contrary to the common belief that equities provide a natural hedge against inflation risk, empirical studies have proven otherwise. With the exception of Boudoukh and Richardson (1993) whose analysis is based on two centuries of data, the overwhelming literature on this subject (Bodie, 1976; Fama and Schwert, 1977; Fama, 1981; Geske and Roll, 1983; Lee, 1992) has come to a consensus that there exists, both on an ex-post and ex-ante basis, a negative correlation between the stock returns and inflation rate, at least over the short to medium term. Moreover, Brown et al. (2001) present evidence that even over longer horizons, the inflation risk hedging property of US stocks and long-term bonds is limited. In a more recent paper, Attié (2009) studies the effectiveness of hedging inflation risk with a portfolio of traditional assets such as cash, bonds, stocks, and commodities. He arrives at the same conclusion that such "hedges" are imperfect at best, and in most cases fail to work at all, even under strategic asset allocation. Similarly, Ang et al. (2012) examine the inflation hedging ability of individual stocks from various sectors, and find that while certain stocks may exhibit significant positive correlation with inflation ex-post, they have little forecasting capability on an ex-ante basis.

This presents an even greater challenge in today's market environment as the consumers worldwide are facing accelerating inflation and slowing economic growth. According to a recent retirement survey by the Society of Actuaries (2011), 71% of preretirees and 58% of retirees in the US are concerned with the ability to keep the value of their savings and investments up with rising inflation. High inflation alone is not the problem; the issue lies in the negative real returns, i.e., inflation rising at a faster pace than the increase in wages and growth in asset returns, over an extended period of time. While pure inflation-protected bonds are effective at providing inflation-indexed income, they are typically low-yielding compared to other asset classes. An alternative is a product whose return is linked to some high-yielding risky assets such as stocks, but floored by the inflation rate over the investment horizon. This allows the investors to participate in the stock market growth while the initial investment is protected in *real* dollar terms. From a retirement planning perspective, this is a more meaningful payout design compared to most of those of the existing Variable Annuities and Equity-Indexed Annuities, where the minimum guaranteed payout is set at a predetermined fixed rate. Unlike the standard inflation-indexed derivatives that are based on a single inflation-linked underlying such as inflation bonds or forwards (Jarrow and Yildirim, 2003; Mercurio, 2005; Hinnerich, 2008; Kruse, 2011), these types of *hybrid* financial instruments with underlyings from more than one asset class are typically challenging to price, even with Monte-Carlo simulations, due to the many stochastic and correlated state variables involved.

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The main contribution of this paper is to show that, under a no-arbitrage Heath–Jarrow–Morton (HJM) type of framework where interest rate term structures are assumed to be Gaussian, closed form pricing results can be obtained for these exotic structures.

The paper is structured as follows. The next section outlines an arbitrage free multi-asset economy for nominal bonds, inflation-linked bonds, and other risky assets, where the term structures follow an HJM model. Section 3 presents two inflation-linked annuity payout designs and their analytical pricing formulas, as well as hedging and sensitivity analysis. Finally, Section 4 concludes the paper.

2. The economy

We expand the Jarrow and Yildirim economy, which is an arbitrage free framework for inflation and interest rate term structures, by generalizing the term structures to a multi-factor HJM model and adding a risky asset. This is akin to the general asset pricing framework in Amin and Jarrow (1992). Suppose we have a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and trading interval $[0, \tau]$, $\tau < \infty$, where the filtration $\{\mathcal{F}_t : t \in [0, \tau]\}$ is generated by $m = 2d+2$ Brownian motions

$$\boldsymbol{W}(t) = \big(W_{n,1}(t), \ldots, W_{n,d}(t), W_{r,1}(t), \ldots, W_{r,d}(t), W_I(t), W_S(t)\big)'.$$

We denote

$$\boldsymbol{W}_n(t) = \big(W_{n,1}(t), \ldots, W_{n,d}(t)\big)'$$

and

$$\boldsymbol{W}_r(t) = \big(W_{r,1}(t), \ldots, W_{r,d}(t)\big)'$$

to be the underlying risks that drive the nominal and real term structures, respectively, while $W_I(t)$ and $W_S(t)$ are drivers of inflation rate and the return on the risky asset, respectively. We assume that the Brownian motions within $\boldsymbol{W}_n$ and $\boldsymbol{W}_r$ are uncorrelated, i.e., $dW_{n,i}(t)dW_{n,j}(t) = 0$ and $dW_{r,i}(t)dW_{r,j}(t) = 0$ for all $i \neq j$, $i, j = 1, 2, \ldots, d$, but we allow for correlations among $\boldsymbol{W}_n$, $\boldsymbol{W}_r$, W_I and W_S.

For any $T \in (0, \tau]$ and $t \in [0, T]$, let $f_n(t, T)$ and $f_r(t, T)$ be the *nominal* and *real* instantaneous forward rates, respectively, at time t on a riskless loan that begins at time T and matures an instant later. We assume an HJM Gaussian economy where the forward rates have the following evolution.

Assumption 1. For a fixed $T \in (0, \tau]$ and $k \in \{n, r\}$ where n and r represent the nominal and real rates, respectively, the T-maturity forward rate at time t evolves as

$$f_k(t, T) = f_k(0, T) + \int_0^t \mu_k(u, T)du + \int_0^t \Xi_k(u, T)' d\boldsymbol{W}_k(u), \quad t \in [0, T] \tag{1}$$

where $\mu_k(t, T)$ is an $\mathcal{F}_t$-adapted random process that satisfies

$$\int_0^T |\mu_k(u, T)|\, du < \infty \quad \text{a.s.}$$

and $\Xi_k(t, T) = \big(\sigma_{k,1}(t, T), \ldots, \sigma_{k,d}(t, T)\big)'$ is a $d \times 1$ vector of deterministic functions of time with

$$\int_0^T \sigma_{k,i}^2(u, T)du < \infty, \quad i = 1, 2, \ldots, d.$$

For $k \in \{n, r\}$, we denote

$$r_k(t) = f_k(t, t)$$

as the corresponding spot rate at time t. Let $I(t)$ be the benchmark inflation index, say the Consumer Price Index (CPI),

$$B_n(t) = \exp\left\{\int_0^t r_n(u)du\right\}$$

be the nominal money market account value measured in dollars, and

$$B_r(t) = \exp\left\{\int_0^t r_r(u)du\right\}$$

the (fictive) real money market account measured in CPI units at time t. The two conditions stated in Assumption 1 ensure that the money market account values $B_n(t)$ and $B_r(t)$ are processes of bounded variation. Let

$$P_k(t, T) = \exp\left\{-\int_t^T f_k(t, u)du\right\}, \quad k \in \{n, r\}$$

be the time-t price of a (nominal or real) zero-coupon bond maturing at time T. Note that while $P_n(t, T)$ is the price of a nominal zero-coupon bond that pays out *one dollar* at time T and a traded asset in the market, $P_r(t, T)$ is the price of a real zero-coupon bond that pays *one CPI unit* at maturity and this price is not directly observable in the economy.

We define the price of a *real* zero-coupon bond *in dollars* as

$$P_r^*(t, T) = I(t)P_r(t, T), \tag{2}$$

at time t. One can think of $P_r^*(t, T)$ as the price of a zero-coupon inflation indexed bond maturing at T, such as the stripped Treasury Inflation-Protected Securities (TIPS) in the US, where the final payment at maturity is the initial principal amount indexed to $I(T)$. We also introduce a new asset

$$B_r^*(t) = I(t)B_r(t)$$

which we define as the inflation-linked real money market account measured in dollars. This asset is assumed to be one of the traded assets in the Jarrow and Yildirim model, although Hinnerich (2008) argues that since this is a fictive asset and does not actually exist in any market, it should not be considered a priori. Though not actually used in any of our pricing formulas presented herewith, we have included it in our model to complete the market.

We now state the stochastic dynamic of the inflation index $I(t)$.

Assumption 2. The stochastic process of the inflation index is given by

$$\frac{dI(t)}{I(t)} = \mu_I(t)dt + \sigma_I(t)dW_I(t) \tag{3}$$

where the drift $\mu_I(t)$ is an $\mathcal{F}_t$-adapted random process with

$$E\left[\int_0^\tau \mu_I(u)^2 du\right] < \infty$$

and the volatility $\sigma_I(t)$ is a deterministic function of time subject to the condition

$$\int_0^\tau \sigma_I(u)^2 du < \infty.$$

With Assumptions 1 and 2, we can show the stochastic dynamic processes of the nominal and inflation-linked bonds.

Corollary 3. *For any $T \in (0, \tau]$, under the physical measure $\mathcal{P}$, the stochastic processes of $P_n(t, T)$, $P_r^*(t, T)$ and $B_r^*(t)$ are given as, respectively:*

$$\frac{dP_n(t, T)}{P_n(t, T)} = a_n(t, T)dt + \tilde{\Xi}_n(t, T)' d\boldsymbol{W}_n(t), \tag{4}$$

$$\frac{dP_r^*(t, T)}{P_r^*(t, T)} = a_r^*(t, T)dt + \sigma_I(t)dW_I(t) + \tilde{\Xi}_r(t, T)' d\boldsymbol{W}_r(t), \tag{5}$$

$$\frac{dB_r^*(t)}{B_r^*(t)} = [\mu_I(t) + r_r(t)]\, dt + \sigma_I(t)dW_I(t) \tag{6}$$

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