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Optimal dividends and ALM under unhedgeable risk[☆]

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HIGHLIGHTS

- We derive optimal investment decisions for insurance companies under unhedgeable risk.
- We study the trade off between the optimal hedge and the fully diversified portfolios.
- We show how to price unhedgeable risk.
- We derive the distribution of the time of bankruptcy.

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ABSTRACT

In this paper we develop a framework for optimal investment decisions for insurance companies in the presence of (partially) unhedgeable risk. The perspective that we choose is from an insurance company that maximises the stream of dividends paid to its shareholders. The policy instruments that the company has are the dividend policy and the investment policy. Using stochastic control theory, we derive simultaneously the optimal investment policy and the optimal dividend policy, taking the insurance risks to be given. We study the trade off between investing in the optimal hedge portfolio and the fully diversified portfolio. We show next how the pricing of unhedgeable risk can also be embedded in our framework. Finally, we derive the distribution of the time of bankruptcy and demonstrate its usefulness in calibrating the model.

1. Introduction

Insurance companies are faced with risks of many types. These include financial risks such as risks inherent in the investment process, but also non-financial risks such as the insurance claims that are at the core of the insurance operation. While most financial risks are generally assumed to be hedgeable, which means that such risks can be replicated in the financial markets, insurance claims are generally considered to be unhedgeable as no replicating portfolio exists for most "insurance events".

In this paper we aim to develop a framework for optimal investment decisions for insurance companies in the presence of (partially) unhedgeable risks. The perspective that we choose is

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from an insurance company that tries to maximise the stream of dividends paid to its shareholders. The policy instruments that the company has to this end are the dividend policy and the investment policy. The insurance company can continue to pay dividends until bankruptcy, and hence the time of bankruptcy is also endogenously controlled by the dividend and investment policies.

The problem of optimising dividends payout schemes has a long history in actuarial mathematics; see, for example, the early contributions by De Finetti (1957), Borch (1967, 1969), Bühlmann (1970) and Gerber (1972, 1979). More recently the study of the problem has received an important impulse by the application of controlled diffusion techniques; see, for example, Paulsen and Gjessing (1997) and the overview paper by Taksar (2000).

The starting points for this paper are the results obtained in the papers by Asmussen and Taksar (1997, AT hereafter), Højgaard and Taksar (1999, HT99 hereafter) and Højgaard and Taksar (2004, HT04 hereafter). Especially the results of HT99 and HT04 are quite interesting. In HT99, they analyse the case where an insurance company searches both an optimal dividend policy and an optimal level of reinsurance. In HT04, they consider the case where also the investment risk can be controlled.

This paper asks a different, yet basic question which appears to have been overlooked in the optimal dividends literature: how does the presence of partially hedgeable and partially unhedgeable risk (which is the usual case in insurance) impact the optimal investment and optimal dividend policies? To answer this question we distinguish carefully between hedgeable risks (i.e., risks that are traded on the financial markets) and unhedgeable risks (i.e., risks that are not traded on the financial markets). This distinction allows us to study the fundamental trade off between investing in the optimal hedge portfolio (reducing risk exposure) and investing in the fully diversified portfolio (increasing expected asset returns). This trade off is at the core of Asset–Liability Management (ALM).

Given our setup, we can use our results to infer what price should be charged for accepting additional unhedgeable risks such that the value of the insurance company remains unchanged. This provides a novel mechanism for the valuation of unhedgeable risks which can be viewed as the marriage of equivalent utility valuation on the one hand, and value and dividend optimisation in ruin theory on the other. We also derive the non-trivial probability distribution of the time of bankruptcy, and we illustrate how this information can be used to calibrate the model such that the implied default probabilities are consistent with observed default probabilities for insurance companies.

The outline of this paper is as follows. In Section 2 we introduce our framework. In Section 3 we derive the optimal policies and we illustrate the derived solution by means of an example. Section 4 discusses the pricing of insurance and Section 5 studies the time of bankruptcy. Section 6 analyzes the optimisation problem under general utility functions and, finally, Section 7 contains some concluding remarks.

2. Stylised insurance company

We fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration $(\mathcal{F}_t)_{t\geq 0}$, which we assume to satisfy *the usual assumptions* (completed and right-continuous). This filtration represents the flow of information on which decisions are based. All Brownian motions that we consider below are defined on $(\Omega, \mathcal{F}, \mathbb{P})$ and adapted to its filtration.

The surplus S_t of the insurance company is defined as the difference in value between assets and liabilities. The insurance company remains solvent as long as $S_t > 0$. We propose to model the surplus process by

$$dS_t = (\boldsymbol{\alpha}'\boldsymbol{\mu}_A + m)dt + \begin{pmatrix} \boldsymbol{\alpha} \\ -1 \end{pmatrix}' \begin{pmatrix} \boldsymbol{\Sigma}_A & \boldsymbol{\sigma}_{AM} \\ \boldsymbol{\sigma}'_{AM} & \sigma_M^2 \end{pmatrix}^{1/2} \times \begin{pmatrix} d\boldsymbol{W}_A \\ dW_M \end{pmatrix} - \sigma_I dW_I. \tag{2.1}$$

To explain this elaborate model for the surplus process we focus first on its liability component, and then on its asset component.

We assume that the liability component is driven by two sources of risk: the diffusion term $\sigma_I dW_I$ which represents the insurance risks, and the diffusion term $\sigma_M dW_M$ which represents the financial market risk component of the liabilities. Many types of insurance liabilities, for example unit-linked or participating contracts, have exposure to financial market risk. We will assume (without loss of generality) that the standard Brownian motions W_M and W_I are independent. The drift term of the surplus process contains a (positive) margin m that the insurance company has built into its liability process to cover the insurance risks and management fees. We assume that there is competition in the insurance market and that m is exogenously given and not a control variable for the insurance company. Please note that the constants m, σ_M and σ_I are absolute quantities and not "percentages".

We assume that the assets of the insurance company can only be invested in financial markets. However, the insurance company can choose from a universe of N investment categories. The $(N\times 1)$-vector $\boldsymbol{\mu}_A$ denotes the vector of expected investment returns, the $(N \times N)$-matrix $\boldsymbol{\Sigma}_A$ denotes the covariance matrix of the investment returns (which means that $\boldsymbol{\Sigma}_A$ is symmetric) and $\boldsymbol{W}_A$ is an N-dimensional standard Brownian motion. We assume that $\boldsymbol{\Sigma}_A$ is a positive definite matrix so that it is non-singular. The vector $\boldsymbol{\alpha}$ captures the exposure in absolute terms to each of the N investment categories. Please note that the constants $\boldsymbol{\mu}_A$ and $\boldsymbol{\Sigma}_A$ denote a vector of return percentages and a matrix of return variances, respectively.

In Eq. (2.1) we have stacked the $N+1$ sources of financial market risk together in an $(N + 1)$-vector, with an $(N + 1) \times (N + 1)$ covariance matrix. The $(N \times 1)$-vector $\boldsymbol{\sigma}_{AM}$ denotes the covariance of each asset category with the insurance liability portfolio. When the financial risk of the insurance liabilities is spanned by the N investment opportunities, then the vector $\boldsymbol{\sigma}_{AM}$ is collinear with the matrix $\boldsymbol{\Sigma}_A$ and as a consequence the $(N+1) \times (N+1)$ covariance matrix is rank-deficient. In this case it will be possible to choose a vector $\boldsymbol{\alpha}$ such that all financial risk drivers are eliminated. This is known as the *replicating portfolio*. In this case the surplus process reduces to $dS_t = mdt - \sigma_I dW_I$. This means that when the insurance company decides to invest in the replicating portfolio, the surplus process is driven by pure insurance risks only. The optimal dividend policy for this special case is investigated in the AT paper.

If the insurance company decides to deviate from the replicating portfolio then the surplus process may benefit from additional excess returns, but at the cost of increased risk. It is this risk/return trade off which is the subject of so-called ALM (Asset–Liability Management) models.

To lighten the notation for the analysis of the surplus process, we replace the $N + 2$ Brownian motions by a single diffusion term which has the same law:

$$dS_t = \left(\boldsymbol{\alpha}'\boldsymbol{\mu}_A + m\right) dt + \left(\begin{pmatrix} \boldsymbol{\alpha} \\ -1 \end{pmatrix}' \begin{pmatrix} \boldsymbol{\Sigma}_A & \boldsymbol{\sigma}_{AM} \\ \boldsymbol{\sigma}'_{AM} & \sigma_M^2 \end{pmatrix} \times \begin{pmatrix} \boldsymbol{\alpha} \\ -1 \end{pmatrix} + \sigma_I^2 \right)^{1/2} dW. \tag{2.2}$$

For a typical insurance company the surplus S is a factor 10–20 smaller than the total asset portfolio A (or the liability portfolio