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A bivariate shot noise self-exciting process for insurance

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HIGHLIGHTS

- Studying a bivariate shot noise self-exciting process.
- Analysing this process' theoretical distributional properties.
- Applying this process to insurance premium calculations.
- Showing that this process can be used for the modelling of catastrophic losses.

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ABSTRACT

In this paper, we study a bivariate shot noise self-exciting process. This process includes both externally excited joint jumps, which are distributed according to a shot noise Cox process, and two separate self-excited jumps, which are distributed according to the branching structure of a Hawkes process with an exponential fertility rate, respectively. A constant rate of exponential decay is included in this process as it can play a role as the time value of money in economics, finance and insurance applications. We analyse this process systematically for its theoretical distributional properties, based on the piecewise deterministic Markov process theory developed by Davis (1984), and the martingale methodology used by Dassios and Jang (2003). The analytic expressions of the Laplace transforms of this process and the moments are presented, which have the potential to be applicable to a variety of problems in economics, finance and insurance. In this paper, as an application of this process, we provide insurance premium calculations based on its moments. Numerical examples show that this point process can be used for the modelling of discounted aggregate losses from catastrophic events.

1. Introduction

A catastrophic event such as flood, storm, hail, bushfire, earthquake or terrorism brings about huge losses in properties, motor vehicles and from the interruption of businesses. Particular examples concern losses due to 2012 Hurricane Sandy, 2011 Great Eastern Japan Earthquake and Tsunami, 2010–2011 Queensland floods, 2009 Victorian Bushfires (Report of 2009 Victorian Bushfires Royal Commission, 2010), 2005 Hurricane Katrina (Burton and Hicks, 2005) and 2001 September 11 attack (Makinen, 2002). The US oil disaster in the Gulf of Mexico in 2010 is another perfect illustration of the magnitude of the shocks that have to be absorbed. These are extreme risks which pose a challenge to insurers and their financial viability and hence improved models are required to predict losses arising from catastrophic events.

To this effect, in this paper we introduce a bivariate shot noise self-exciting process, which has both externally excited joint jumps following a shot noise Cox process and two separate self-excited jumps, which themselves are Hawkes processes. This process accommodates the clustering arrival of losses observed in practice due to increases in frequency and intensity of natural and man-made disasters.

Self-exciting (or Hawkes) processes (Hawkes, 1971; Hawkes and Oakes, 1974; Daley and Vere-Jones, 2003) are versatile point processes, interesting both from a theoretical as well as a practical point of view. The theoretical foundation of Hawkes processes can be traced from a series of papers written by Brémaud and Massoulié (1996, 2001, 2002) and Liniger (2009). Relevant publications in seismology and the modelling of the occurrence of earthquakes are Vere-Jones (1975), Adamopoulos (1976), Vere-Jones (1978), Ozaki (1979), Vere-Jones and Ozaki (1982) and Ogata (1988).

The applications and modelling of Hawkes processes in finance can be found in Chavez-Demoulin et al. (2005), McNeil et al. (2005), Bauwens and Hautsch (2009), Bowsher (2007), Aït-Sahalia et al. (2010) and Embrechts et al. (2011). Credit default modelling using

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these processes can be noticed in Errais et al. (2010) and Giesecke and Kim (2011). Stabile and Torrisi (2010) applied a Hawkes process in an insurance context studying the asymptotic behaviour of infinite and finite horizon ruin probabilities.

More recently Dassios and Zhao (2011) introduced a dynamic contagion process, which is a generalisation of the externally excited Cox process with shot noise intensity and the self-excited Hawkes process applying to credit risk. Dassios and Zhao (2012) also examined infinite horizon ruin probability with its Monte Carlo simulation using this process as the claim arrival process. These aforementioned papers are the univariate models. In contrast we extend further to quantify collateral losses that would occur more often due to global warming, climate change and terrorism using a bivariate shot noise self-exciting process.

This paper is structured as follows. In Section 2, we provide a cluster process representation to define and characterise a bivariate shot noise self-exciting process. To do so, we adopt the one used by Dassios and Zhao (2011). In Section 3.1, we analyse this process systematically for its theoretical distributional properties, based on the piecewise deterministic Markov process theory developed by Davis (1984), and the martingale methodology used by Dassios and Jang (2003). The joint moment of two processes and its covariance are derived in Section 3.2, where the first and second moment of each component of the process are given by the ones proposed by Dassios and Zhao (2011). As an application of this process, we provide insurance premium calculations based on these quantities in Section 4. Section 5 discusses our research findings with further possible research.

2. Definition

Bivariate shot noise self-exciting processes include both *externally excited joint* jumps, which are distributed according to a Cox process with shot noise intensity, and two separate *self-excited* jumps, which are distributed according to the branching structure of a Hawkes process with an exponential fertility rate, respectively. We provide the mathematical definition for our process in Definition 2.1 as a cluster point process, additionally characterised by its infinitesimal generator. Many alternative definitions of a bivariate shot noise self-exciting process can be given. In this paper, we adopt the one used by Dassios and Zhao (2011), where they gave a cluster process representation for a dynamic contagion process.

Definition 2.1. Each point in a bivariate shot noise self-exciting process is one of two types: an immigrant process occurring jointly or two separate offspring processes. The cluster centres of $\mathbb{D}$ are the particular points called *immigrants*, where $\mathbb{D}$ is a cluster point process on $\mathbb{R}_+$, the other points are called *offspring*. They have the following structure.

(1) The immigrants are distributed according to a Cox process A with joint points $\binom{D_m^{(1)},}{D_m^{(2)}}$ in $(0,\infty)$, $m \in \mathbb{N}_0$ and shot noise stochastic intensity processes, i.e.

$$a^{(1)} + \left(\lambda_0^{(1)} - a^{(1)}\right) e^{-\delta^{(1)}t} + \sum_{i\geq 1} X_i^{(1)} e^{-\delta^{(1)}(t-T_{1,i})} \mathbb{I}\left(T_{1,i} \leq t\right),$$

$$a^{(2)} + \left(\lambda_0^{(2)} - a^{(2)}\right) e^{-\delta^{(2)}t} + \sum_{i\geq 1} X_i^{(2)} e^{-\delta^{(2)}(t-T_{1,i})} \mathbb{I}\left(T_{1,i} \leq t\right),$$

where $a^{(d)} > 0$, $d = 1, 2$ are constant reversion levels, $\lambda_0^{(d)} > 0$, $d = 1, 2$ are constants as the initial values of a bivariate shot noise self-excited process (defined later by (2.1)), $\delta^{(d)} > 0$, $d = 1, 2$ are constant rates of exponential decay, $\left\{X_i^{(1)}, X_i^{(2)}\right\}_{i=1,2,\ldots}$ is a sequence of independent identical distributed positive externally excited *joint* jumps with distribution function $F\left(x^{(1)}, x^{(2)}\right)$, $x^{(1)} > 0$, $x^{(2)} > 0$ at the corresponding random times $\left\{T_{1,i}\right\}_{i=1,2,\ldots}$ following a homogeneous Poisson process N_t with constant intensity $\rho > 0$ and $\mathbb{I}$ is the indicator function.

(2) Each immigrant $D_m^{(1)}$ generates a cluster $C_m^{(1)}$, which is the random set formed by the points of generations $0, 1, 2, \ldots$ with the following branching structure. The immigrant D_m is said to be of generation 0. Given generations $0, 1, 2, \ldots, r$ in $C_m^{(1)}$, each point $T_{2,j} \in C_m^{(1)}$ of generation r generates a Cox process on $\left(T_{2,j}, \infty\right)$ of offspring of generation $r + 1$ with intensity function $Y_j e^{-\delta(\cdot - T_{2,j})}$, where a positive self-excited jump Y_j at time $T_{2,j}$ has distribution function $G(y)$, $y > 0$, independent of the points of generation $0, 1, 2, \ldots, r$.

(3) Each immigrant $D_m^{(2)}$ generates a cluster $C_m^{(2)}$, which is the random set formed by the points of generations $0, 1, 2, \ldots$ with the following branching structure. The immigrant D_m is said to be of generation 0. Given generations $0, 1, 2, \ldots, r$ in $C_m^{(2)}$, each point $T_{2,k} \in C_m^{(2)}$ of generation r generates a Cox process on $\left(T_{2,k}, \infty\right)$ of offspring of generation $r + 1$ with intensity function $Z_k e^{-\delta(\cdot - T_{2,k})}$, where a positive self-excited jump Z_k at time $T_{2,k}$ has distribution function $H(z)$, $z > 0$, independent of the points of generation $0, 1, 2, \ldots, r$.

(4) Given the immigrants, the centred clusters

$$C_m^{(d)} - \binom{D_m^{(1)},}{D_m^{(2)}} = \left\{ T_{\substack{2,j(\text{for } d=1)\\ 2,k(\text{for } d=2)}} - \binom{D_m^{(1)},}{D_m^{(2)}} : T_{\substack{2,j(\text{for } d=1)\\ 2,k(\text{for } d=2)}} \in C_m^{(d)} \right\},$$
$$D_m \in A,$$

are independent identically distributed, and independent of A.

(5) $\mathbb{D}$ consists of the union of all clusters, i.e.

$$\mathbb{D} = \cup_{\substack{m\in\mathbb{N}_0\\ d=1,2}} C_m^{(d)}.$$

Therefore, a bivariate shot noise self-exciting process is given by

$$\begin{aligned}
\lambda_t^{(1)} &= a^{(1)} + \left(\lambda_0^{(1)} - a^{(1)}\right) e^{-\delta^{(1)}t} + \sum_{i\geq 1} X_i^{(1)} e^{-\delta^{(1)}(t-T_{1,i})} \\
&\quad \times \mathbb{I}\left(T_{1,i} \leq t\right) + \sum_{j\geq 1} Y_j e^{-\delta^{(1)}(t-T_{2,j})} \mathbb{I}\left(T_{2,j} \leq t\right), \\
\lambda_t^{(2)} &= a^{(2)} + \left(\lambda_0^{(2)} - a^{(2)}\right) e^{-\delta^{(2)}t} + \sum_{i\geq 1} X_i^{(2)} e^{-\delta^{(2)}(t-T_{1,i})} \\
&\quad \times \mathbb{I}\left(T_{1,i} \leq t\right) + \sum_{k\geq 1} Z_k e^{-\delta^{(2)}(t-T_{2,k})} \mathbb{I}\left(T_{2,k} \leq t\right),
\end{aligned} \tag{2.1}$$

where $\left\{Y_j\right\}_{j=1,2,\ldots}$ is a sequence of independent identically distributed positive self-excited jumps with distribution function $G(y)$, $y > 0$, at the corresponding random times $\left\{T_{2,j}\right\}_{j=1,2,\ldots}$, $\left\{Z_k\right\}_{k=1,2,\ldots}$ is another sequence of independent identically distributed positive self-excited jumps with distribution function $H(z)$, $z > 0$, at the corresponding random times $\left\{T_{2,k}\right\}_{k=1,2,\ldots}$ and the sequence $\left\{X_i^{(1)}, X_i^{(2)}\right\}_{i=1,2,\ldots}$, $\left\{T_{1,i}\right\}_{i=1,2,\ldots}$ $\left\{Y_j\right\}_{j=1,2,\ldots}$ and $\left\{Z_k\right\}_{k=1,2,\ldots}$ are assumed to be independent of each other.

The immigrants and offspring can be considered as "main shocks/jumps/arrivals" and "after shocks/jumps/arrivals", respectively. This leads to a various applications in economics, finance and insurance.

From the definition above and because of the exponential decays, we can see that $\left(\lambda_t^{(1)}, \lambda_t^{(2)}\right)$ is a Markov process. $\lambda_t^{(d)}$ ($d = 1, 2$) decrease with rate $\delta^{(d)}\left(\lambda_t^{(d)} - a^{(d)}\right)$ and incur additive upward externally excited joint jumps that have distribution function $F\left(x^{(1)}, x^{(2)}\right)$ with rate ρ, and two additive upward self-excited