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On the mortality/longevity risk hedging with mortality immunization

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HIGHLIGHTS

- Model-/magnitude-free mortality durations and convexities are defined and derived.
- Duration/convexity matching strategies are proposed, classified and compared.
- The VaR and hedge effectiveness are used to evaluate hedging performances.
- The weights of a portfolio are determined by duration/convexity matching strategies.
- The matching strategies can significantly hedge the mortality/longevity risks.

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ABSTRACT

In this paper, we define the mortality durations and convexities of the prices of life insurance and annuity products with respect to an instantaneously proportional change and an instantaneously parallel shift, respectively, in μs (the forces of mortality), ps (the one-year survival probabilities) and qs (the one-year death probabilities), and further derive them as magnitude-free closed-form formulas. Then we propose several duration/convexity matching strategies to determine the weights of two or three products in an insurance portfolio. With the stochastic mortality models, we evaluate the Value-at-Risk (VaR) values and the hedge effectiveness of the surpluses at time zero for the underlying portfolio with these matching strategies. Illustrated numerical examples demonstrate that the duration/convexity matching strategies with respect to an instantaneously proportional change in μs and qs can significantly hedge the mortality/longevity risks.

1. Introduction

Interest rate immunization whereby the value of a portfolio will be little affected in response to a change in interest rates has been studied and applied widely in hedging interest-linked securities. For example, Bierwag (1977) shows that the optimal selection of an immunized bond portfolio is related to random shifts in the term structure of interest rates. Hilliard (1984) constructs a minimum variance hedge by adding a portfolio of financial futures to the spot portfolio of assets and liabilities. Duration matching, one of the approaches for interest rate immunization, is widely used in asset and liability management to help match liabilities with assets in order to stabilize cash flows in the future. Developments and uses of duration in bond portfolio management can be found in Bierwag et al. (1983), Chance (1990), Longstaff and Schwartz (1995), Panjer (1998), and Poitras and Jovanovic (2007).

In the same manner, the premium and reserve of a life insurance or annuity policy are also affected by interest rates; numerous studies have made a lot of efforts to study interest rate immunization in the liabilities of life insurance policies for insurers. Redington (1952) demonstrates the first- and second-order conditions ensuring that when a constant change in the force of interest is made, the resulting present value of the net cash flows at time zero will be equal to or larger than that without the change in interest rates. The first-order condition is that the duration of the present value of the net cash flows is zero, or that the duration of the present value of the cash inflows matches that of the cash outflows—the so-called duration matching strategy in finance. Fisher and Weil (1971) study this immunization problem for a single liability and relax Redington's assumption of a constant force of interest. Shiu (1987, 1988, 1990) extends the Fisher–Weil's immunization theorem by assuming that the constant change in the force of interest is a function of time as well and further study the multiple-liability immunization problem. The literature has analyzed the durations and convexities of life insurance liabilities; see, e.g., Briys and de Varenne (1997), Santomero and Babbel (1997), Courtois and Denuit (2007), and Tsai (2009).

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Nowadays, insurers issuing life insurance and annuity policies not only bear interest rate risk but also face another huge threat from mortality and longevity risks. Over the past few decades, mortality rates have been displaying a dramatic improvement. Such an improvement could lead to the possibilities of financial distress or insolvency for annuity providers, retirement programs and social security systems. One of the solutions to hedging longevity/mortality risks is to build effective mortality models in order to provide accurate mortality rates for the prices of life insurance and annuity policies. The model proposed by Lee and Carter (1992) is the most widely cited and used method in mortality prediction and applications. The CBD model proposed by Cairns et al. (2006) is another broadly used model. A variety of extensions of the two models have been made; see, e.g., Haberman and Renshaw (2009), Li et al. (2009), Plat (2009), and Cox et al. (2010).

Alternatively, mortality-linked securities (e.g., longevity bonds, q-forward, survivor swaps, annuity futures, mortality options, and survivor caps) can be used for hedging longevity and mortality risks. A growing literature regarding the designs and prices of these securities can be found in, for example, Blake and Burrows (2001), Lin and Cox (2005), Dowd et al. (2006), Menoncin (2008), and Stevens et al. (2010). A mortality-catastrophe bond (called Vita) was issued by Swiss Re for the first time in December 2003. In December 2010, Swiss Re issued a longevity-trend bond (called Kortis). Survivor (or longevity) swaps have been issued by insurance companies and investment banks since 2007. A number of pension funds have recently started hedging longevity risks with these financial products, in particular survivor swaps. Using longevity bonds, Tsai et al. (2011) propose an asset–liability management strategy to hedge the aggregate risk of annuity providers under the assumption that both the interest rate and mortality rate are stochastic. Coughlan et al. (2011) develop a framework for analyzing the longevity basis risk and hedge effectiveness when using an index-based longevity instrument. Cairns (in press) constructs minimum variance hedges using q-forward or deferred longevity swaps and demonstrates the hedge effectiveness of the strategies with and without the inclusion of recalibration risk, parameter uncertainty and Poisson risk. Cairns et al. (in press) decompose the key risk factors influencing the effectiveness of longevity hedges.

Unlike interest rate immunization and other approaches to hedging mortality/longevity risks, mortality rate immunization has still attracted relatively little attention. Mortality immunization gives life insurers and annuity providers natural hedging opportunities through the proper allocation of life insurance and annuity policies. Cox and Lin (2007) show that the natural hedge potential that arises from combining life annuities and death benefits may be substantial. Tsai et al. (2010) minimize CVaR (conditional value at risk) values to determine the allocation of insurance products. Duration/convexity matching is the most common used strategy in mortality immunization. Wang et al. (2010) and Plat (2011) adopt the effective mortality duration by assuming a proportional change in μs (the forces of mortality) and qs (one-year death probabilities), respectively, to determine the weights of two life insurance and annuity policies in a portfolio, which together with the resulting weights all depend on the magnitude of the proportional change. However, there are very few articles regarding mortality duration and convexity due to the lack of their formal definitions. Li and Hardy (2011) and Li and Luo (2012) define the measure so called key q-duration which is a variation of the effective duration and then construct a longevity hedge with q-forward contracts given the measure. Tsai and Jiang (2011) define the mortality durations of the prices of life and annuity products with respect to each of the two parameters in the linear transform of the force of mortality. Tsai and Chung (2013) derive magnitude-free closed-form formulas for the mortality durations and convexities with respect to a proportional movement and a parallel shift, respectively, in the force of mortality.

Here we are interested in further developing duration/convexity matching strategies for mortality immunization. The mortality rates in calculating the premiums and reserves of life and annuity policies can be expressed in terms of μs (the forces of mortality), ps (the one-year survival probabilities), or qs (the one-year death probabilities). The premiums/reserves move in response to a change in the underlying mortality rates. The change may be constant/parallel, proportional, a mixture of both, or of some pattern. For example, to set aside more funds some life insurance companies adopt $(1+c)\cdot q$s for reserving where qs are used for pricing life insurance and annuity products and c is a constant. Thus, we define mortality durations and convexities with respect to an instantaneously proportional change and an instantaneously parallel shift, respectively, in μ, p and q, and provide an innovative approach to deriving them as magnitude-free closed-form formulas. To facilitate hedging, we propose several duration/convexity matching strategies for determining the weights of two or three products in an insurance portfolio with respect to μ, p and q, and evaluate the hedging performances of these strategies by comparing some risk measures. Besides, we extend the theorem in Tsai and Chung (2013) regarding the feasibility of some three-product portfolios using these strategies.

The remainder of this paper proceeds as follows. In Section 2, we introduce our proposed mortality durations and convexities. In Section 3, we demonstrate the numerical mortality durations and convexities of the surpluses at time zero for some life and annuity products calculated with the forecasted mortality rates from the Lee–Carter model. Sections 4 and 5 propose mortality duration/convexity strategies for determining the product weights in two-product and three-product insurance portfolios and compare the VaRs and the hedge effectiveness of the simulated surpluses at time zero for the portfolios based on the weights resulting from these strategies. Section 6 presents the conclusion.

2. Mortality durations and convexities

Duration is a cornerstone of the strategy for interest immunization. Macaulay duration, modified duration and dollar duration are three common types of durations in finance, which measure the sensitivity of the price of an asset to a parallel shift in the interest rate. Convexity measures the curvature or second derivative of the price of an asset that varies with a constant change in the interest rate. In this section, we first review the definitions of traditional duration and convexity with respect to the interest rate.

Let $P(\delta)=\sum_{k=1}^{n}C_k\times e^{-\delta\cdot k}$ be the price of a financial security at time 0 with cash flows C_k at time k, $k=1,\ldots,n$, where $\delta=\ln(1+i)$ is the force of interest and i is the interest rate. The durations are used to measure the sensitivity of the price $P(\delta)$ with respect to a parallel shift in δ. The traditional modified duration with respect to δ is defined by

$$MD_\delta[P(\delta)]=-\frac{\partial P(\delta)}{\partial\delta}\cdot\frac{1}{P(\delta)}=\frac{1}{P(\delta)}\sum_{k=1}^{n}k\times C_k\times e^{-\delta\cdot k},$$

and the dollar duration with respect to δ is defined as

$$DD_\delta[P(\delta)]=-\frac{\partial P(\delta)}{\partial\delta}=\sum_{k=1}^{n}k\times C_k\times e^{-\delta\cdot k}$$
$$=P(\delta)\cdot MD_\delta[P(\delta)].$$

The classical convexity, measuring how the duration of the price $P(\delta)$ changes as a constant movement in δ, is defined by

$$MC_\delta[P(\delta)]=\frac{\partial^2P(\delta)}{\partial\delta^2}\cdot\frac{1}{P(\delta)}=\frac{1}{P(\delta)}\sum_{k=1}^{n}k^2\times C_k\times e^{-\delta\cdot k}.$$