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Analytical valuation of periodical premiums for equity-linked policies with minimum guarantee

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HIGHLIGHTS

- We consider the problem of computing periodical premiums of equity-linked policies with minimum guarantee.
- A simple analytical formula is proposed to compute the fair periodical premium.
- Numerical results confirm that the proposed model computes accurate values for all the considered cases.

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ABSTRACT

We consider the problem of computing fair periodical premiums of equity-linked policies with a minimum guarantee. The policy payoff at maturity may be decomposed into two components: a fixed part representing the guaranteed payment and a European call option written on the equity reference fund. The deemed periodical contributions into the reference fund may be considered as negative dividends paid by the reference fund and the fair value of the policy may be derived through a closed-form formula by mimicking the valuation of an option written on an underlying security that pays fixed dividends. Numerical results show that the proposed model computes accurate values.

1. Introduction

An equity-linked policy is a contract that allows a policyholder to link the policy payoff to the performance of a reference portfolio made up of equities of the same kind. To mitigate the risk of possible losses, a minimum guarantee is usually inserted into the contract. In the simplest possible case, the policyholder pays at the inception a single premium and receives at maturity the greater between the terminal value of the initial investment and the minimum guarantee. The final payoff may be decomposed into two parts: a fixed amount representing the value of the guarantee and a European call option on the reference fund with strike price given by the guarantee. If one assumes an evaluation framework as that proposed by Black and Scholes (1973) to evaluate financial options, a closed-form formula is readily available for the value of the single premium.

In the case of periodical premiums, things are more complicated because at each premium payment date the reference fund value jumps up due to the contribution deemed into the reference fund and no analytical formula is available. Brennan and Schwartz (1976) and Boyle and Schwartz (1977) proposed a finite difference approach while Delbaen (1986) computed fair periodical premiums through Monte Carlo simulations. Bacinello and Ortu (1993) analyzed the case of periodical premiums with endogenous guarantees.

In this setting, we propose an analytical formula for the periodical premium of an equity-linked policy with a minimum guarantee. We assume that a fixed component of each periodical premium is invested into the reference fund whose evolution is described by a piecewise lognormal model with upward jumps of magnitude equal to the deemed contributions. Ignoring mortality risk, the policy pays off at maturity the greater between the reference fund value and the minimum guarantee. The policy terminal payoff can be obtained as the combination of two components: a fixed amount representing the minimum guarantee and a European call option written on the reference fund with a strike price given by the minimum guarantee. Hence, the policy value at inception can be calculated as the sum of the value of the minimum guarantee and of the call option. The first term is readily available by discounting at the risk-free interest rate the guaranteed amount at maturity. To evaluate the call option, we may observe that the underlying asset resembles a stock that pays fixed (negative) dividends. The problem of evaluating financial options on stocks paying fixed dividends has been widely addressed. The simplest approach is the so-called escrowed model

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which assumes that the underlying asset price minus the present value of all dividends to be paid during the option lifetime follows a geometric Brownian motion under the risk-neutral probability measure. Then, by substituting this modified initial price to the current underlying asset price, the Black–Scholes formula furnishes the desired option value. This naïve approach may produce consistently biased option values. This is due essentially to the fact that the entire volatility surface is shifted upward at each dividend payment date and, as a consequence, the absolute volatility is too big in the period before a dividend is paid. In order to overcome this drawback, different approaches have been proposed by adjusting the volatility in a consistent manner so that the Black–Scholes formula may still be applied. Among these, Bos, Gairat and Shepeleva (BGS henceforth) (Bos et al., 2003), using perturbation theory, proposed a simple and effective model to solve the option pricing problem. Following the same approach, we derive a closed-form formula for the fair value of periodical premiums of an equity-linked policy with a minimum guarantee. Numerical results highlight that the proposed model computes accurate values in all the considered cases. The rest of the paper is organized as follows. In Section 2, we deal with the problem of deriving an analytical approximation of the periodical premium for an equity-linked policy with minimum guarantee. In Section 3, we illustrate numerical results of the proposed model. In Section 4, we draw conclusions.

2. Evaluating the fair periodical premium

We consider an equity-linked policy issued at time $t = 0$ with maturity T years. Without considering mortality, the policyholder agrees to pay n constant premiums, P, at the beginning of n equally-spaced time intervals of length $\Delta t = T/n$, such that each premium payment date is $t_i = i\Delta t,\ i = 0, \ldots, n-1$. At each premium payment date, a fixed component, D, of the periodical premium is invested into a reference fund made up of equities of the same kind. At the contract maturity, the policy pays off the maximum between the reference fund value and a guaranteed amount that, without loss of generality, we set equal to

$$G(T) = \sum_{i=0}^{n-1} De^{g(T-t_i)} = D\frac{e^{g(T+\Delta t)} - e^{g\Delta t}}{e^{g\Delta t} - 1},$$

where $g > 0$ is the minimum guaranteed force of interest.[1] In other words, the insurer is forced to pay at maturity at least the fixed deemed contributions accrued at rate g. Labeling $F(t)$ the reference fund value at time t, the policy payoff at maturity is given by

$$\max(F(T), G(T)).$$

The terminal value of the policy payoff may be decomposed as $G(T) + C(T)$, the value of the minimum guarantee plus the payoff at maturity, $C(T) = \max(F(T) - G(T), 0)$, of a European call option written on the reference fund with strike price equal to the minimum guarantee. A different possible decomposition of the policy payoff at maturity is $F(T) + P(T)$, the reference fund value plus the payoff, $P(T) = \max(G(T) - F(T), 0)$, of a European put option written on the reference fund with strike price equal to the minimum guarantee. In order to compute the fair policy value at inception we consider the call decomposition but the same approach could be used to compute the fair policy value by working with the put decomposition.

We assume an evaluation framework with a complete, frictionless market without arbitrage opportunities. This implies the existence of a unique equivalent martingale measure, Q, and that each contingent claim must be evaluated by discounting at the risk-free interest rate, r, the expectation, under Q, of the contingent claim at maturity.

In this setting, we postulate that the evolution of the reference fund value is described by a geometric Brownian motion between premium payment dates with upward jumps of magnitude D at dates where premiums are paid, i.e.,

$$dF(t) = rF(t)dt + \sigma F(t)dW(t),$$
$$t_i < t < t_{i+1},\ i = 0, \ldots, n-1,$$

and

$$F(t_i) = F(t_i^-) + D, \qquad F(0) = D, \tag{1}$$

where σ is the volatility of the reference fund rate of return and $W(t)$ is a standard Brownian motion under the risk-neutral probability measure. The fair policy value at inception may be obtained as the present value of the guaranteed amount plus the value of the call option, i.e.,

$$V_0^Q(G(T) + C(T)) = V_0^Q(G(T)) + V_0^Q(C(T)),$$

where $V_0^Q(\cdot)$ represents the operator computing the value at the inception of a contingent claim under the risk-neutral probability measure. $V_0^Q(G(T))$ is easily computed by discounting at the risk-free interest rate the fixed guarantee $G(T)$, i.e.,

$$V_0^Q(G(T)) = e^{-rT}D\frac{e^{g(T+\Delta t)} - e^{g\Delta t}}{e^{g\Delta t} - 1}.\text{[2]} \tag{2}$$

Computing the call option value at inception is more difficult due to the upward jumps in the reference fund value dynamics.

We propose to compute the fair policy premium through an analytical approximation of the call option. The call option embedded into the contract may be viewed as a European call written on an underlying asset that pays fixed (negative) dividends at each premium payment date. The simplest model for evaluating an option written on a stock paying fixed discrete dividends is the so-called escrowed model. It is based on the idea that, because the future dividends are known in advance, their present value may be added to the current underlying asset price. Then, starting from the adjusted current price, the dynamics of the underlying asset price is described by a geometric Brownian motion without the upward jumps induced at each premium payment date, i.e.,

$$d\overline{F}(t) = r\overline{F}(t)dt + \sigma\overline{F}(t)dW(t),$$
$$\overline{F}(0) = F(0) + \sum_{i=1}^{n-1} De^{-rt_i} = D\frac{1 - e^{-rT}}{1 - e^{-r\Delta t}}. \tag{3}$$

Hence, the option price is computed by applying the Black–Scholes formula using the adjusted current underlying asset price, i.e.,

$$V_0^Q(C(T)) \approx C_{BS}(\overline{F}(0), G(T), r, \sigma, T)$$

with

$$C_{BS}(\overline{F}(0), G(T), r, \sigma, T) = \overline{F}(0)N(d_+) - G(T)e^{-rT}N(d_-),$$

where

$$d_\pm = \frac{1}{\sigma\sqrt{T}}\left[\log\left(\frac{\overline{F}(0)}{G(T)}\right) + \left(r \pm \frac{\sigma^2}{2}\right)T\right],$$

and $N(\cdot)$ is the distribution function of a standard normal random variable. Unfortunately, the escrowed model computes, in general, consistently biased option prices. The reason is that the volatility, $\sigma\overline{F}(t)$, of the "adjusted" underlying asset price process, is different from the volatility, $\sigma F(t)$, of the "true" process and is too big in

[1] If $g = 0$ the minimum guaranteed amount at maturity is nD.

[2] If $g = 0$, $V_0^Q(G(T)) = e^{-rT}nD$.