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A new immunization inequality for random streams of assets, liabilities and interest rates

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HIGHLIGHTS

- Immunization inequality for insurers' surplus under random assets, liabilities and interest rates is proven.
- The resulting lower bound is of high precision for a variety of cases.
- Explicit formulas for portfolios of life liabilities vs streams of net premiums are provided.
- Applications for some well-known models of interest rates are treated.

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ABSTRACT

In this paper, we investigate the problem of immunization of insurers' surplus when liabilities are financed by a stream of assets. The term structure of interest rates is assumed to be random, as are the streams of assets and liabilities. A new inequality for changes in the portfolio surplus in response to changes in the term structure of interest rates is proven. A comparison with other immunization inequalities shows that it gives better lower bounds for a wide variety of scenarios. The inequality is sharp in the sense that the lower bound is attainable for some interest rate perturbations. Whenever net insurance premiums are considered, it is factorized into a product of two terms: one depending only on the change of interest rates, and the other depending only on the portfolio structure. Hence the second term may be treated as a measure of the interest rate risk. We call it $\mathcal{L}^2$-measure, because it is related to the second order distance between assets and liabilities. Explicit formulas for this measure for portfolios of some life products vs streams of net premiums are given. Applications to the Merton's, Vasicek's and simple log-normal models of interest rate are also provided.

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1. Introduction

The paper focuses on the problem how interest rate fluctuations can influence the valuation of portfolios consisting of assets and liabilities. The theory of the portfolio immunization is dating back to the paper by Redington (1952) (see Shiu, 1990, Reitano, 1992, 1996 and Hürlimann, 2002 for more recent development of the theory). It is also known as Asset–Liability Management as it refers to a joint management of the streams of assets and liabilities. In particular, Redington was the first who said that both streams should be valued consistently, using the same structure of interest rates. So let us consider streams of portfolio incomes $\{A_1, \ldots, A_n\}$ and outcomes $\{L_1, \ldots, L_n\}$ due at dates $0 < t_1 < \cdots < t_n$. We assume that the present value of every surplus $S_j = A_j - L_j$ is established using a discount function corresponding to a given basic term structure of interest rates (TSIR). Let v_j denote the present value, at time $t = 0$, of a monetary unit due at $t = t_j$, for any $j \in \{1, \ldots, n\}$. Then $s_j = S_j v_j$ denotes the present value of the surplus S_j. The present value of the whole portfolio is given by:

$$V = \sum_{j=1}^{n} s_j.$$

We are interested in how much the value of the portfolio can change in response to the changes in TSIR. Let $s'_j = S_j v'_j$ denote the present value of the surplus S_j, at time $t = 0$, under the perturbed TSIR. Then:

$$\Delta V = V' - V = \sum_{j=1}^{n} s'_j - \sum_{j=1}^{n} s_j. \quad (1)$$

The primary goal of immunization was to find conditions under which ΔV is nonnegative for any change in TSIR. However, this postulate in general is contradictory with the assumption that arbitrage is not allowed on the market (see Panjer, 1998 for details).

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Therefore Fisher and Weil (1971) gave sufficient conditions for perfect immunization but only for parallel shifts of the interest rate intensity. On the other hand, Fong and Vasicek (1984), Nawalkha and Chambers (1996), Shiu (1990), Gajek et al. (2005) and others give lower bounds on ΔV which may take negative values.

Fong and Vasicek (1984) assume durations of assets and liabilities to be equal. Then the lower bound on (1) is a product of a term depending on the derivative of the interest rate intensity changes and M-Square of the portfolio. The model works well for the scenarios when the interest rates are shifted almost in parallel.

The model of Nawalkha and Chambers (1996) seems to be more general as the assumption of durations equality is excluded. The bound on (1), given by Nawalkha and Chambers (1996), is a product of two terms: one depending on the term structure perturbation and the other being M-Absolute. In contrast with the traditional approach, the M-Absolute model immunizes only partially against the height shift, but it seems to reduce the risk caused by the shifts in the slope, curvature and other term structure shape parameters.

Gajek et al. (2005) gave the bound on (1) which was based on a different methodology. Instead of bounding the discount factors from below, they used the Cauchy–Schwarz inequality. It enabled to separate the changes in TSIR from the term structure of assets and liabilities. In Section 2 we improve their inequality and extend it to the case when all the interest rates, assets and liabilities are stochastic processes. We give a lower bound on (1) of the following form:

$$E\left(V'-V\right) \geq \frac{1}{n}EV \cdot \sum_{j=1}^{n} Ef_j - \mathcal{L}^2(\mathbf{s}) \cdot \mathcal{L}^2(\mathbf{f}), \tag{2}$$

where $\mathbf{s} = (s_1, \ldots, s_n)$, $\mathbf{f} = (f_1, \ldots, f_n)$ with $f_j = \frac{v_j'}{v_j} - 1$ for every $j \in \{1, \ldots, n\}$ and $\mathcal{L}^2(\mathbf{x}) = \left(\sum_{j=1}^{n} E\left(x_j - \frac{1}{n}\sum_{j=1}^{n} Ex_j\right)^2\right)^{\frac{1}{2}}$ for $\mathbf{x} = (x_1, \ldots, x_n)$. The interest rates changes are allowed to be measured only at the moments of payments, not in a continuous time regime. It seems to reduce the overvalued role of short time perturbations in TSIR in the approaches of Fong and Vasicek (1984) and Nawalkha and Chambers (1996).

Portfolio immunization under additional restrictions on the portfolio structure was treated by Gajek (2005).

In Section 3 we calculate the $\mathcal{L}^2$-measure for some well-known interest rates models. In Section 4 a more detailed comparison of our approach with the ones given by Fong and Vasicek (1984), Nawalkha and Chambers (1996) and Gajek et al. (2005) is presented. In particular, it is shown via numerical examples, that for some scenarios of interest rate changes it works better than the bounds known in the literature. In Section 5 applications to some typical insurance products are provided.

2. The main result

Let us notice that:

$$\Delta V = V' - V = \sum_{j=1}^{n} s_j' - \sum_{j=1}^{n} s_j = \sum_{j=1}^{n} s_j f_j, \tag{3}$$

where $f_j = \frac{v_j'}{v_j} - 1$ for every $j \in \{1, \ldots, n\}$.

We assume that the interest rates, assets and liabilities are stochastic processes, hence the surplus s_j' is a random variable for every $j \in \{1, \ldots, n\}$. The same concerns (1), and one can look for a lower bound on its expected value. Throughout this paper we will assume that f_j, A_j and L_j have finite second moments for every $j \in \{1, \ldots, n\}$ which is a technical assumption that imposes no restrictions on applications of the model in practice.

Theorem 2.1. *Let us denote* $\mathbf{s} = (s_1, \ldots, s_n)$ *and* $\mathbf{f} = (f_1, \ldots, f_n)$. *Under the above assumptions, the following inequality holds:*

$$E\left(V'-V\right) \geq \frac{1}{n}EV \cdot \sum_{j=1}^{n} Ef_j - \mathcal{L}^2(\mathbf{s}) \cdot \mathcal{L}^2(\mathbf{f}), \tag{4}$$

where $\mathcal{L}^2(\mathbf{x}) = \left(\sum_{j=1}^{n} E\left(x_j - \frac{1}{n}\sum_{j=1}^{n} Ex_j\right)^2\right)^{\frac{1}{2}}$ *for* $\mathbf{x} = (x_1, \ldots, x_n)$.

Proof. For any real λ and arbitrary random variables a_j, b_j, $j \in \{1, \ldots, n\}$ the following equality holds:

$$E\sum_{j=1}^{n} a_j b_j = E\sum_{j=1}^{n}\left(a_j - \frac{1}{n}\bar{a}\right)\left(b_j - \lambda\right) + \frac{1}{n}\bar{a}\bar{b}, \tag{5}$$

where $\bar{a} = E\sum_{j=1}^{n} a_j$ and $\bar{b} = E\sum_{j=1}^{n} b_j$.

Applying identity (5) with $a_j = s_j$ and $b_j = f_j$ and observing that $\sum_{j=1}^{n} a_j = V$ we get:

$$E\left(V'-V\right) = E\sum_{j=1}^{n} s_j f_j = E\sum_{j=1}^{n}\left(s_j - \frac{1}{n}EV\right)\left(f_j - \lambda\right) + \frac{1}{n}EV \cdot \sum_{j=1}^{n} Ef_j.$$

By the Cauchy–Schwarz inequality for expected values we get:

$$E\left(V'-V\right) \geq -\sum_{j=1}^{n}\left[E\left(s_j - \frac{1}{n}EV\right)^2\right]^{\frac{1}{2}}\left[E\left(f_j - \lambda\right)^2\right]^{\frac{1}{2}} + \frac{1}{n}EV \cdot \sum_{j=1}^{n} Ef_j.$$

Now, applying the Cauchy–Schwarz inequality for sums, we obtain:

$$E\left(V'-V\right) \geq -\left[\sum_{j=1}^{n} E\left(s_j - \frac{1}{n}EV\right)^2\right]^{\frac{1}{2}}\left[\sum_{j=1}^{n} E\left(f_j - \lambda\right)^2\right]^{\frac{1}{2}} + \frac{1}{n}EV \cdot \sum_{j=1}^{n} Ef_j. \tag{6}$$

Since λ is an arbitrary real number we can choose it so that the right side of inequality (6) is maximized. Observe that the lower bound in (6) is maximal when the function

$$g(\lambda) = \sum_{j=1}^{n} E\left(f_j - \lambda\right)^2$$

attains its minimum. This holds for $\lambda = \frac{1}{n}E\sum_{j=1}^{n} f_j$. With this choice of lambda, the result is proven. □

Theorem 2.1 can be easily adapted to the case when the time horizon for the portfolio immunization is H (possibly greater than 0). Indeed, let us notice that:

$$\Delta V_H = V_H' - V_H = \frac{\sum_{j=1}^{n} s_j'}{v_H'} - \frac{\sum_{j=1}^{n} s_j}{v_H} = \sum_{j=1}^{n} s_{j,H} \cdot f_{j,H},$$

where $s_{j,H} = \frac{A_j v_j - L_j v_j}{v_H}$, $f_{j,H} = \frac{v_H}{v_H'} \cdot \frac{v_j'}{v_j} - 1$.

The following theorem can be proven analogously to Theorem 2.1.

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