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Optimal reinsurance in the presence of counterparty default risk

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HIGHLIGHTS

- The optimal reinsurance in the presence of counterparty default risk is found.
- An elegant and constructive procedure is provided for convex distorted risk measures.
- The effect of reinsurance over the policyholder welfare is discussed.

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ABSTRACT

The optimal reinsurance arrangement is identified whenever the reinsurer counterparty default risk is incorporated in a one-period model. Our default risk model allows the possibility for the reinsurer to fail paying in full the promised indemnity, whenever it exceeds the level of regulatory capital. We also investigate the change in the optimal solution if the reinsurance premium recognises or not the default in payment. Closed form solutions are elaborated when the insurer's objective function is set via some well-known risk measures. It is also discussed the effect of reinsurance over the policyholder welfare. If the insurer is Value-at-Risk regulated, then the reinsurance does not increase the policyholder's exposure for any possible reinsurance transfer, even if the reinsurer may default in paying the promised indemnity. Numerical examples are also provided in order to illustrate and conclude our findings. It is found that the optimal reinsurance contract does not usually change if the counterparty default risk is taken into account, but one should consider this effect in order to properly measure the policyholders exposure. In addition, the counterparty default risk may change the insurer's ideal arrangement if the buyer and seller have very different views on the reinsurer's recovery rate.

1. Introduction

Two parties are involved in a standard reinsurance contract: the *insurer*, *cedent*, *insurance buyer*, or even simpler, *buyer*, who has an interest in transferring part of its risk to the *reinsurer*, also known as *insurance seller*, or even simpler *seller*. Let $X \geq 0$ be the total amount that the insurer is liable to pay during the duration of the insurance contract, with the distribution function denoted by $F(\cdot)$ and survival function $\bar{F}(\cdot) = 1 - F(\cdot)$. In addition, the right end-point $x_F := \inf\{z \in \Re : F(z) = 1\}$ of the loss distribution could be either finite or infinite, even though the finiteness assumption would be more realistic. The reinsurance seller agrees to pay, $R[X]$, the amount by which the entire loss exceeds the insurer amount, $I[X]$. Thus, $I[X] + R[X] = X$. There are many possible reinsurance arrangements, which depend on the particular choice of the insurer and reinsurer sharing the premiums and underwritten risks. For example, the liabilities are shared in a fixed proportion under *proportional* reinsurance and therefore $I[X] = cX$, where $c \in [0, 1]1$ is a constant. Another common arrangement is the *stop-loss* reinsurance contract, for which the buyer retained loss is limited to a fixed amount, M, known as *retention limit*. The net amount paid by the insurer is therefore given by $\min\{X, M\} := X \wedge M$.

There is a vast literature on identifying the optimal risk transfer contract between two insurance companies within a one-period setting. The first attempts are attributed to Borch (1960) and Arrow (1963) where the expected utility is maximised. Further extensions have been developed for various decision criteria that depend on the risk measure choice (for example, see Van Heerwaarden et al., 1989, Young, 1999, Kaluszka, 2001, 2005, Verlaak and Beirlant, 2003, Kaluszka and Okolewski, 2008, Ludkovski and Young, 2009). Two commonly used in practice risk measures, *Value-at-risk (VaR)* and *Expected Shortfall (ES)*, are considered by Cai et al. (2008), Cheung (2010) and Chi and Tan (2011). The classical risk model setting has been successfully studied in the literature by Centeno and Guerra (2010) and Guerra and Centeno (2008, 2010), where

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a natural choice for optimisation is the maximisation of the adjustment coefficient.

The classical approach of finding the optimal risk sharing ignores the possibility of default in payment that the risk transfer initiator is exposed to, known as the *counterparty default risk*. This can be viewed as a special case of the *background risk*, under the additive background risk assumption. This setting has been investigated by Dana and Scarsini (2007) via the expected utility maximisation. The paper of Biffis and Millossovich (2012) is related to the latter work and analyses the effect of counterparty default risk under some economic constraints. Bernard and Ludkovski (2012) deal with the same problem, but the loss given default is considered loss-dependent. Our approach is different, in the sense that the insurer prefers a risk measure when making a decision of sharing the liability. In addition, the default is assumed to be endogenous as it has been seen in Biffis and Millossovich (2012), and the seller's available assets are given by its regulatory capital. That is, the loss of basic own funds which the insurer would incur if the insurance seller defaults, known as the *loss given default*, is assumed to be proportional to the excess of the indemnity over the reinsurer level of capital requirements. Like any other counterparty default risk model, there are pros and cons for our choice, and we believe that our model is sufficiently rich to provide an understanding of the change in the optimal arrangement if the buyer incorporates the reinsurer chance to default.

As it has been anticipated, the reinsurer's default in payment is assumed to occur whenever the indemnity exceeds the reinsurer capital. Motivated by the *Solvency* II regulatory requirements developed within the European insurance industry, where the risk exposures are measured via VaR, we assume that the seller operates in an environment that is VaR regulated. The latter assumption enables us to identify the solution of an infinite dimensional optimisation problem by imposing mild and economically sound restrictions for the set of possible risk transfers. Alternatively, one may commit to a specific class of reinsurance contracts, such as focusing only on the stop-loss arrangements, which allows one to solve standard finite dimensional optimisation problems.

The VaR of a generic loss variable Z at a confidence level a, $VaR_a(Z)$, represents the minimum amount of capital that makes the insurance company to be solvent at least a% of the time. The mathematical formulation is then given by

$$VaR_a(Z) := \inf\{z \geq z_0 : \Pr(Z \leq z) \geq a\},$$

where $z_0 := \sup\{z \in \Re : \Pr(Z \leq z) = 0\}$ represents the left end point of the distribution of Z. By convention, $\inf \varnothing = +\infty$ is true.

In the absence of default risk, the indemnity is $R[X]$, otherwise it is given by

$$\tilde{R}[X;\delta] := R[X] \wedge VaR_\beta\big(R[X]\big) + \delta\Big(R[X] - VaR_\beta\big(R[X]\big)\Big)_+,$$

where $\delta \in [0, 1]$ represents the *recovery rate* used to calculate the loss given default. By definition, $(z)_+ = \max\{z, 0\}$. Note that the seller is assumed to be VaR-regulated, and for example, $\beta = 99.5\%$ whenever Solvency II is in force. Thus, the *probability of default* is $1 - \beta$, i.e. the insolvency probability allowed by the regulator. Obviously, the no-default scenario is recovered if we set $\delta = 1$.

The seller and buyer may have different beliefs about the recovery rate, but it is likely that the insurer to be more pessimistic than the reinsurer. In this paper, it is assumed that $0 \leq \delta_1 \leq \delta_2 \leq 1$, where δ_1 and δ_2 are the recovery rates of the buyer and seller, respectively. Let $\mathbf{P}\big(\tilde{R}[X;\delta_2]\big)$ be the reinsurance premium if the seller incorporates the default risk. Therefore, the total insurer loss becomes

$$\tilde{\mathbf{L}}[X;\delta_1,\delta_2] := X - \tilde{R}[X;\delta_1] + \mathbf{P}\big(\tilde{R}[X;\delta_2]\big).$$

A large class of such risk measures is given by

$$\varphi(Z) := \int_0^1 VaR_s(Z)\Phi(s)\,ds = \int_0^\infty g\big(\Pr(Z > z)\big)\,dz - \int_{-\infty}^0 \Big(g\big(\Pr(Z > z)\big) - 1\Big)\,dz, \tag{1.1}$$

where $\Phi(s) = g'(1 - s)$. This class is known as the *distorted* (see Wang and Young, 1998 and Jones and Zitikis, 2003) and *spectral* (see Acerbi, 2002) class of risk measures, respectively. Note that the distorted function $g : [0, 1] \to [0, 1]$ is assumed to be non-decreasing and concave such that $g(0) = 0$ and $g(1) = 1$. Therefore, $g(\cdot)$ is differentiable almost everywhere, but not necessarily differentiable on [0, 1]. Consequently, using the usual derivative $g'(\cdot)$, whenever it exists, does not change the representation from (1.1) (for further details, see Dhaene et al., 2012).

The previously-mentioned class includes the well-known ES risk measure, which has various representations in the literature (see Acerbi and Tasche, 2002). We only refer to the next definition:

$$\mathrm{ES}_\alpha(Z) := \frac{1}{1-\alpha}\int_\alpha^1 VaR_s(Z)\,ds = VaR_\alpha(Z) + \frac{1}{1-\alpha}\mathbf{E}\big(Z - VaR_\alpha(Z)\big)_+.$$

Interestingly, this risk measure is a special case of the Haezendonck–Goovaerts class, which was introduced many years ago by Haezendonck and Goovaerts (1982). Further details can be found in Bellini and Rosazza Gianin (2012), Goovaerts et al. (2004, 2012) and the references therein.

We aim to identify the optimal arrangement that reduces the seller's risk as much as possible, where the risk is evaluated via VaR or a distorted risk measure. VaR and ES are standard tail risk measures used in practice to set technical provisions and capital requirements, and therefore, it is natural to believe that both are good choices for the insurance company to base its decision. That is, we intend to minimise $\varphi_I\big(\tilde{\mathbf{L}}[X;\delta_1,\delta_2]\big)$ over a set of feasible reinsurance contracts, where $\varphi_I(\cdot)$ represents a measure of the risk taken by the insurer. In order to avoid potential moral hazard issues related to the reinsurance arrangement, the set of feasible contracts is given by

$$\mathcal{F} := \big\{R(\cdot) : I(x) = x - R(x) \text{ and } R(x) \text{ are non-decreasing functions}\big\}.$$

Note that $R \in \mathcal{F}$ implies that I and R are 1-Lipschitz functions, i.e. $|I(y) - I(x)| \leq |y - x|$ and $|R(y) - R(x)| \leq |y - x|$ are true for all $x, y \geq 0$.

The premiums are usually assumed to be positively loaded, and therefore it is expected to have that $\mathbf{P}\big(\tilde{R}[X;\delta_2]\big) \geq \mathbf{E}\big(\tilde{R}[X;\delta_2]\big)$. A common insurance pricing is the *expected value* principle, $\mathbf{P}\big(\tilde{R}[X;\delta_2]\big) = (1+c)\mathbf{E}\big(\tilde{R}[X;\delta_2]\big)$, where $c > 0$ is known as the *security loading factor*. Our results are given for a more general pricing method, where the seller prices the premium based on a distorted risk measure. That is, $\mathbf{P}\big(\tilde{R}[X;\delta_2]\big) = (1+c)\varphi_R\big(\tilde{R}[X;\delta_2]\big)$, where $\varphi_R(\cdot)$ is a distorted risk measure. Besides its general formulation, distorted risk measures have proved to be valuable choices for insurance pricing (see for example, Wang, 2000).

The rest of the paper is organised as follows. Section 2 investigates the VaR-based decisions, while Section 3 evaluates optimal arrangements based on distorted risk measures. Section 4 explains the effect of reinsurance over the policyholder welfare. The last section provides some numerical examples that are meant to illustrate our main results and conclude the paper.

2. VaR-based optimal reinsurance contract

The current section describes the optimal choice for the buyer if VaR measures its perception about risk. In other words, we assume