Insurance: Mathematics and Economics 53 (2013) 722–732

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Conditional copula simulation for systemic risk stress testing

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HIGHLIGHTS

- Propose copula methods for systemic risk analysis of financial institutions.
- Develop methodology for stress testing the financial market using copulas.
- Derive new procedures for conditional simulation of Archimedean and vine copulas.
- Analyze and stress test CDS spreads of 38 major international institutions.

ARTICLE INFO

Article history:
Received March 2013
Received in revised form
July 2013
Accepted 13 September 2013

Keywords:
Multivariate copula
Sampling
Vine copula
Systemic risk
Stress testing
CDS spreads

ABSTRACT

Since the financial crisis of 2007–2009 there is an active debate by regulators and academic researchers on systemic risk, with the aim of preventing similar crises in the future or at least reducing their impact. A major determinant of systemic risk is the interconnectedness of the international financial market. We propose to analyze interdependencies in the financial market using copulas, in particular using flexible vine copulas, which overcome limitations of the popular elliptical and Archimedean copulas. To investigate contagion effects among financial institutions, we develop methods for stress testing by exploiting the underlying dependence structure. New approaches for Archimedean and, especially, for vine copulas are derived. In a case study of 38 major international institutions, 20 insurers and 18 banks, we then analyze interdependencies of CDS spreads and perform a systemic risk stress test. The specified dependence model and the results from the stress test provide new insights into the interconnectedness of banks and insurers. In particular, the failure of a bank seems to constitute a larger systemic risk than the failure of an insurer.

1. Introduction

Dealing with the lessons learned from the financial crisis, the discussion about systemic risk has become more and more important. The collapse of Lehman Brothers in 2008 showed that the sudden and uncontrolled breakdown of a global financial company not only affected other financial institutions and seriously endangered the stability of the global financial sector but also had a great impact on the real economy of several countries around the world. As a result, the Financial Stability Board (FSB) developed guidelines to assess the systemic importance of financial institutions, markets, and instruments. The FSB defines systemic risk as "*the risk of disruption to financial services that is* (i) *caused by an impairment of all or parts of the financial system and* (ii) *has the potential to have serious negative consequences for the real economy*" (Financial Stability Board, 2011). Furthermore, an institution, market, or instrument is regarded as systemic if "*its failure or malfunction causes widespread distress, either as a direct impact or as a trigger for broader contagion*" on the financial system and/or the real economy.

The systemic relevance of an institution can be assessed based on several criteria that have been identified by the FSB. The three most important are size, lack of substitutability, and interconnectedness: Financial institutions whose "*distress or disorderly failure, because of their size, complexity, and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity*" (Financial Stability Board, 2011) are called systemically important. These institutions will face additional regulatory measures to reduce the systemic risk imposed by them. The Basel Committee on Banking Supervision (2011) and the International Association of Insurance Supervisors (2012) have developed methodologies to determine globally systemically important banks and insurers, respectively. The assessment methodology for insurers differs to that used for banks, since it takes into account the fundamental differences in the business models of banks and insurance companies. While a systemic classification of insurers has not been published yet, a list of globally systemically important banks is released on a yearly basis. In 2012, there were 28 banks on this list (Financial Stability Board, 2012).

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http://dx.doi.org/10.1016/j.insmatheco.2013.09.009

Despite the popular expression "too big to fail", it has been argued in recent literature that the interconnectedness of an institution is much more important in the assessment of systemic risk: Cont and Moussa (2010) and Cont et al. (2013) find that the impact of the failure of an institution strongly depends on the interdependencies among institutions and less on its size. Similarly, Markose et al. (2012) observe in their analysis of interconnectedness in the US banking sector that only a few major institutions play a dominant role in terms of network centrality and connectivity. With respect to contagion in the US insurance sector, Park and Xie (2011) evaluate the impact of reinsurer downgradings on US property–casualty insurers and conclude that a systemic crisis caused by reinsurance transactions is rather unlikely. Billio et al. (2012) analyze the interdependencies among financial institutions from different sectors using principle component analysis and Granger causality networks and detect an interesting asymmetry in the financial system, as banks are more likely to transmit shocks than insurers, hedge funds or brokers/dealers. Hence, in light of this research, it is more appropriate to speak of systemically important institutions as "too (inter-)connected to fail".

The exploration of contagion and interconnectedness is also the topic of this article. We propose to use copulas to analyze interdependencies in the global financial market, notably in the banking as well as in the insurance sector and not in both sectors in isolation, as it is often done. In doing so, we aim to find out whether there are significant differences in the dependence structure among banks and among (re-)insurers. As a statistical tool for dependence modeling, copulas allow for an accurate analysis beyond linear correlations and common multivariate Gaussian distributions. Therefore, we not only consider the popular classes of Archimedean and elliptical copulas, but also the more recently proposed vine copulas (see Kurowicka and Joe, 2011 and Czado et al., 2013 for recent overviews). Such vine copulas allow to take into account tail and asymmetric dependencies and therefore overcome limitations of the elliptical copulas that are typically used in larger dimensional dependence analysis. Vine copulas may also provide more parsimonious parameterizations of multivariate distributions and therefore constitute useful models for a flexible dependence analysis (see also Brechmann and Czado, forthcoming).

Stress testing is an important tool for the assessment and classification of systemic risk. The systemic relevance of an institution is decisively determined by the potential impact of its failure on other institutions. It is therefore crucial to analyze such stress situations in the market by taking into account the interdependence among the institutions. Statistically speaking, we are interested in the following situation: Let $\boldsymbol{X} := (X_1, \ldots, X_d)'$ be a random vector of risk quantities. Then we are interested in the case $\boldsymbol{X}_{-i}|X_i = x_i$, $i \in \{1, \ldots, d\}$, where $\boldsymbol{X}_{-i}$ denotes the random vector $\boldsymbol{X}$ without the ith component and the event $\{X_i = x_i\}$ corresponds to a stress situation. For instance, let X_i be the company value, then a stress situation occurs when x_i is very small. Clearly, such an analysis requires the availability of the conditional distribution of $\boldsymbol{X}_{-i}|X_i = x_i$, given the specific underlying dependence model. As this distribution is typically not known in closed form, conditional simulation algorithms are needed for the scenario analysis. While these are straightforward and well known in the case of elliptical copulas, we derive appropriate methods for Archimedean and vine copulas.

The methodology developed in this article is used in a case study of 38 important financial institutions from all over the world, among them 20 insurers and 18 banks. Their credit default swap spreads, as market-based indicators of the credit worthiness, are statistically analyzed and appropriate multivariate dependence models are constructed. A stress testing exercise then provides insights into the systemic relevance of the different institutions. We detect differences among regional markets and, in addition, among the banking and the insurance sector. Interestingly, the classification of globally systemically important banks is hardly reflected in the data. Furthermore, the analysis reveals new results regarding the classification of insurers, which, however, cannot yet be compared to an official classification.

The remainder of the paper is structured as follows. Section 2 provides the necessary methodological background on copulas and vine copulas in particular. Conditional copula simulation for the classes of elliptical, Archimedean and vine copulas is then treated in Section 3. The case study is presented in Section 4. Section 5 concludes.

2. Copulas

The statistical notion of dependence is closely related to the concept of copulas. In the first place, a d-dimensional copula simply is a multivariate distribution function on $[0, 1]^d$ with uniform marginal distribution functions. According to the theorem by Sklar (1959), any multivariate distribution is directly linked to a copula. Let $\boldsymbol{X} = (X_1, \ldots, X_d)' \sim F$ with marginal distribution functions $F_1, \ldots, F_d$, then Sklar (1959) shows that

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)), \quad (x_1, \ldots, x_d)' \in (\mathbb{R} \cup \{-\infty, \infty\})^d, \tag{2.1}$$

where C is a d-dimensional copula. That is, Sklar's Theorem (2.1) allows to decompose any multivariate distribution in terms of its margins and a copula that specifies the between-variable dependence. If $\boldsymbol{X}$ is a continuous random vector, then the copula C is unique and the multivariate density f of $\boldsymbol{X}$ can be decomposed as

$$f(x_1, \ldots, x_d) = c(F_1(x_1), \ldots, F_d(x_d))f_1(x_1) \ldots f_d(x_d), \tag{2.2}$$

where c is the density of the copula C and $f_1, \ldots, f_d$ are the marginal densities of f. More details on copulas in general can be found in the comprehensive reference books by Joe (1997) and Nelsen (2006). Here, we concentrate on the popular classes of elliptical and Archimedean copulas as well as the more recently proposed vine copulas, which are also known as pair-copula constructions.

If F is an elliptical distribution function (see Fang et al., 1990 and McNeil et al., 2005), then the associated copula C is also called elliptical. More precisely, an elliptical copula is defined through inversion of Sklar's Theorem (2.1) as

$$C(u_1, \ldots, u_d) = F(F_1^{-1}(u_1), \ldots, F_d^{-1}(u_d)), \quad (u_1, \ldots, u_d)' \in [0, 1]^d,$$

where F is elliptical and $F_1, \ldots, F_d$ are the corresponding margin distribution functions. The most popular examples of elliptical copulas are the Gaussian copula with correlation matrix $R = (\rho_{ij})_{i,j=1,\ldots,d} \in [-1, 1]^{d\times d}$ and the Student's t copula with correlation matrix $R \in [-1, 1]^{d\times d}$ and $\nu > 2$ degrees of freedom. In addition to being reflection symmetric (if $\boldsymbol{U} \sim C$, then it also holds that $\mathbf{1} - \boldsymbol{U} \sim C$, where $\mathbf{1} := (1, \ldots, 1)'$), the Gaussian copula is tail independent, while the Student's t copula exhibits symmetric lower and upper tail dependence (Embrechts et al., 2002).

Another important class of copulas are Archimedean copulas. For a generator function $\varphi : [0, 1] \to [0, \infty)$ with $\varphi(1) = 0$ and d-monotone inverse φ^{-1} (see McNeil and Nešlehová, 2009) a d-dimensional Archimedean copula is defined as

$$C(u_1, \ldots, u_d) = \varphi^{-1}\left(\varphi(u_1) + \cdots + \varphi(u_d)\right), \quad (u_1, \ldots, u_d)' \in [0, 1]^d. \tag{2.3}$$

Popular Archimedean copulas are the Clayton, Gumbel and Frank copulas, which possess properties different to those of the elliptical copulas, such as asymmetric tail dependence. From