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Valuation and risk assessment of disability insurance using a discrete time trivariate Markov renewal reward process

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HIGHLIGHTS

- New approach for the valuation and risk assessment of disability insurance.
- Modeling of possible dependences between durations in two successive states.
- Identification of potential relevance depending on dependency and model setup.

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ABSTRACT

In disability insurance, the impact of the duration since the inception of disability on future recovery and mortality rates has been modeled by bivariate Markov renewal processes and the associated semi-Markov process, but these processes do not incorporate potential dependences between the durations in two successive states. Thus, the aim of this paper is to introduce a discrete time trivariate Markov renewal reward model, an associated formula for higher moments and a corresponding simulation that include the potential dependence between the durations, i.e. the inter-arrival times, in two successive states. The proposed model is compared with two alternative models that do not include this dependence.

1. Introduction

Disability insurance has gained an increasingly global relevance. Since the reform of the German statutory pension program in 2001 for example, the German social security system only covers a reduction in earning capacity but no longer disabilities for those who were born after January 1st, 1961 (see Graf von der Schulenburg, 2005). As a result, individual disability insurance is regarded as being existential (see Schwark, 2011). In the economic analysis executed by Chandra and Samwick (2005), precautionary saving has been identified as being less useful as a hedge against disability risk and the value of disability insurance policies has been emphasized. Therefore, a thorough risk analysis of disability insurance policies is becoming increasingly important. In previous literature, the dependence between the time elapsed since the inception of disability on the one hand and recovery and mortality rates on the other hand has been observed (see, e.g., Gregorius, 1993, Rupprecht, 1990, Segerer, 1993) and modeled by discrete time bivariate Markov renewal processes and the associated semi-Markov processes for instance. These processes do not incorporate a potential dependence between the durations, i.e. the inter-arrival times, in two successive states though. Thus, the aim of this paper is to extend literature by proposing a trivariate Markov process, which includes this dependence, and by providing an associated formula and a simulation approach to analyze prospective reserves.

A large part of the literature dealing with disability insurances concerns itself with the future development of these policies and possible dependencies. It has been noted that a healthier lifestyle, more advanced medical technologies as well as safety practices have resulted in a decline of work-limiting disability (see Chandra and Samwick, 2005, Wade and Liebenberg, 2012) and Schwark (2011) states that the lower frequency of claims as well as the

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improved calculability have resulted in less expensive policies. Several criteria have been identified to be significant for the premium calculation: Besides the objective criteria age, gender and occupation, personal criteria such as the biometric risk, the general economic situation, the degree of carefulness and the attractiveness of benefits have been detected while the latter implicitly impose a moral risk (see, e.g., Bull, 1980, Doudna, 1977, Kolster et al., 1998, Malik, 2011 and Rupprecht, 1990). It must be noted that the premium calculation and the underlying probabilities additionally highly depend on the policy design, which varies in different nations (see, e.g., Gregorius, 1993, Haberman and Pitacco, 1999 and Segerer, 1993). Also, several risks are generally excluded. In Germany, for instance, "risks of war and civil commotion, active participation in criminal acts, intentional self-mutilation, motor racing and flying risks, either professionally or as a sport" are not covered (see Segerer, 1993, p. 133).

In the literature, various approaches have been presented to deal with the different states of a policyholder and their complex interactions. Generally, a three-state-model is applied to reflect the basic states of a policyholder: active, disabled and dead. Depending on the setup, transitions from the disabled state to the active state, which are referred to as recoveries, or virtual transitions from a state to itself are sometimes considered. This three-state-model may be extended by splitting the disabled state in consideration of different disability degrees (see, e.g., Stenberg et al., 2007) or by adding additional states for lapses and pensioners (see, e.g., D'Amico et al., 2009 and Haberman and Pitacco, 1999). One interaction between these different states is the duration of disability, which has been divided in an acute and a chronic phase: While in the first phase, high mortality and recovery rates are observed, the latter usually reports much lower recovery and mortality rates (see, e.g., Kolster et al., 1998, Olivieri and Pitacco, 2009, Pitacco, 2004, Rupprecht, 1990 and Segerer, 1993). This duration effect may be modeled by a split of disability states with respect to the duration since disability inception (see, e.g., Gregorius, 1993, Haberman and Pitacco, 1999, Pitacco, 1995, Pitacco, 2004) or by applying semi-Markov processes (see Haberman and Pitacco, 1999). Other interactions include the dependence of transition probabilities on the duration of the policy and the dependence of these probabilities on the total time spent in each state since policy issue (see, e.g., Haberman and Pitacco, 1999, Olivieri and Pitacco, 2009 and Pitacco, 1995).

Several approaches have been suggested for the valuation of disability insurances. Haberman and Pitacco (1999) provide an overview of existing actuarial approaches for the valuation of disability policies and for the calculation of premiums and reserves. The concept of semi-Markov reward processes is used by Stenberg et al. (2007) to provide a discrete time method with initial backward recurrence time which directly calculates all raw moments of the net present value of future premiums and benefits. Haberman and Pitacco (1999) state the general concept of semi-Markov processes with backward recurrence times and D'Amico et al. (2009) extend this concept by defining discrete time semi-Markov transition probabilities with initial and/or final backward recurrence times and by stating the associated general formulas. They empirically show the importance of these probabilities and the corresponding discrete time semi-Markov reward model with initial and final backward recurrence times is presented in D'Amico et al. (2013b). In contrast, Helwich (2008) defines a continuous time semi-Markov process with initial and final backward recurrence times, states the associated general formula for the transition probabilities and provides a corresponding continuous time semi-Markov reward process with initial backward recurrence times. Limnios and Oprişan (2003) advance the concept of semi-Markov processes by introducing n-th order semi-Markov chains in continuous time, which were used by D'Amico et al. (2013a) to propose a discrete time homogeneous semi-Markov model of order two in state and duration. This semi-Markov model is an extension of the (first order) homogeneous bivariate Markov renewal process since the transition probabilities are not just conditioned on the current state, but also on the previous state and the time spent in that previous state. As a result, this model and the associated reward model stated in D'Amico et al. (2012) include potential dependences between the durations, i.e. inter-arrival times, in two successive states and have been applied to the modeling of wind speed. This potential dependence may also be of importance for disability insurances because the time previously spent in the disability state may not only affect the next health state, but also the time spent in that state. More specifically, active policyholders, who have been disabled for a longer period of time, may be expected to become disabled earlier and more often than those with a shorter duration of the previous disability.

In this paper, we aim to propose an alternative approach to the semi-Markov model in D'Amico et al. (2012, 2013a) in order to incorporate the potential dependence between two successive inter-arrival times in the context of disability insurance. More precisely, we introduce a non-homogeneous trivariate Markov renewal reward process whose transition probabilities are conditioned on the current state and, unlike the homogeneous semi-Markov model of order two in state and duration, on the time spent in the current state as well as the time of the transition into this state. Here, in contrast to the bivariate process, three random variables run together. We follow the approach in Stenberg et al. (2007) and provide a recursive equation for calculating raw moments of the prospective reserve. Additionally, we present a simulation approach, which allows further in-depth analysis based on risk measures and quantiles. Because of the lack of empirical data, we test the new model by assuming potential dependences between the inter-arrival times in two successive states and, then, by comparing the trivariate Markov renewal process with the bivariate Markov renewal process that is associated with the semi-Markov process. Hereby, the obtained results indicate a potential impact of the potential dependencies between the inter-arrival times in two successive states.

The remainder of this paper is structured as follows. In Section 2, the bivariate Markov renewal process and the associated semi-Markov process are presented along with the definitions of rewards. In the following section, the trivariate Markov renewal process is defined, the net present value of the insurance contract is introduced as a random variable and the recursive equation for the n-th moment of the random variable is provided. The simulation technique is introduced in the section "Simulation Approach" while the numerical results are discussed in the section "Numerical Analysis". The last section concludes.

2. Discrete time bivariate Markov renewal reward process

2.1. Definition of the discrete time bivariate Markov renewal process

Let $I = \{1, 2, \ldots, m\}$ be a finite state space where each state may represent a disability level, lapse or death for instance. We follow the notations and definitions in D'Amico et al. (2009) with respect to the discrete time non-homogeneous bivariate Markov renewal process and define the random variables J_n and T_n on a probability space (Ω, Σ, P) as

$$J_n : \Omega \to I \qquad T_n : \Omega \to \mathbb{N}$$

where Ω is the sample space, $\mathbb{N}$ denotes the set of all natural numbers including zero and (J_n, T_n), $n \in \mathbb{N}$, is referred to as a non-homogeneous bivariate Markov renewal process. The natural filtration of this bivariate process is specified as $F_n = \sigma(J_u, T_u; u \leq n)$. The random variable J_n symbolizes the state occupied at the n-th transition whereas T_n stands for the time of the n-th transition. In