Insurance: Mathematics and Economics 52 (2013) 421–434

Contents lists available at SciVerse ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Control variates and conditional Monte Carlo for basket and Asian options

Kemal Dinçer Dingeç *, Wolfgang Hörmann

Department of Industrial Engineering, Boğaziçi University, 34342 Bebek-İstanbul, Turkey

HIGHLIGHTS

- A new simulation method for European basket and Asian options is presented.
- It is based on a new control variate and conditional Monte Carlo.
- It is more efficient than the classical control variate method.

ARTICLE INFO

Article history:
Received December 2011
Received in revised form
February 2013
Accepted 4 March 2013

JEL classification:
G13
C15
C63

Keywords:
Basket options
Asian options
Monte Carlo simulation
Control variate
Conditional Monte Carlo

ABSTRACT

A new, very efficient and fairly simple simulation method for European basket and Asian options under the geometric Brownian motion assumption is presented. It is based on a new control variate method that uses the closed form of the expected payoff conditional on the assumption that the geometric average of all prices is larger than the strike price. The combination of that new control variate with conditional Monte Carlo and quadratic control variates leads to the newly proposed algorithm. Numerical experiments show that the new algorithm is more efficient than the classical control variate method using the geometric price.

1. Introduction

Basket options are popular multivariate derivative securities. Their payoff depends on the weighted average of the underlying asset prices and there exists no closed form solution for the price of basket options. Hence a number of studies emerged that suggest an efficient numerical method for basket options.

Tree methods, PDE based finite difference methods and Fourier transform methods are among the most widely used techniques for option pricing. For one dimensional problems, PDE methods provide a fast solution with quadratic convergence. However, for multivariate options, the computational complexity increases exponentially with respect to the problem dimension. In fact, for dimensions larger than three, Duffy (2006, p. 270) suggests in his monography on PDE methods to use other techniques rather than PDE based finite difference methods due to the curse of dimensionality. Similar problems for increasing dimensions also occur for Fourier transform methods. Thus for higher dimensional options the most practicable method seems to be Monte Carlo simulation. Its speed of convergence is not influenced by the dimension of the problem. In addition, it allows for a simple error bound.

Approximations are fast solution alternatives to the exact methods. There are a number of studies suggesting new approximations or bounds for the price of basket options, see, for instance, Curran (1994), Milevsky and Posner (1998), Ju (2002), Brigo et al. (2004), Deelstra et al. (2004, 2010), Zhou and Wang (2008) and Alexander and Venkatramanan (2012). The disadvantage of the approximations is that the size of the error is unknown and there is no way to reduce it.

The payoff of Asian options depends on the average of the prices of a single asset at different time points. Thus the structure of the payoff is similar to that of basket options. Like for basket options, there exists no closed form solution for the price of Asian options. However, there are some fast techniques special to Asian options. The one dimensional PDE method of Večeř (2001, 2002) and the FFT based convolution method of Černý and Kyriakou (2011) are

* Corresponding author. Tel.: +90 212 359 7077; fax: +90 212 265 1800.
E-mail addresses: kdingec@yahoo.com, kemal.dingec@boun.edu.tr (K.D. Dingeç), hormannw@boun.edu.tr (W. Hörmann).

0167-6687/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.insmatheco.2013.03.002

two important examples. Also, many approximations suggested for basket options can be used or adapted for Asian options. See Boyle and Potapchik (2008) for a recent survey of the methods suggested for Asian options. As mentioned there, Monte Carlo simulation is also well suited for pricing Asian options.

Although simulation is a highly flexible and general method, its efficiency for specific problems depends on exploiting the special features of that problem via variance reduction techniques. The aim in variance reduction is to reduce the variance of the estimator in order to increase the efficiency. Clearly for a fixed error bound smaller variance directly implies a smaller sample size and so smaller computational time. However, although variance reduction methods are not so complicated, their application to financial simulation problems is not trivial. To design a successful variance reduction method, one has to understand the characteristics of the problem of interest.

There are few studies suggesting new variance reduction methods for basket options (e.g. Dahl and Benth, 2001, 2002 and Pellizzari, 2001). On the other hand, Asian options are often used as a test case to exemplify the effectiveness of the general variance reduction methods, see, for example, Glasserman et al. (1999), L'Ecuyer and Lemieux (2000), Guasoni and Robertson (2008), Kawai (2010), Étoré and Jourdain (2010), Étoré et al. (2011), Jasra and Del Moral (2011) and Wang and Sloan (2011). The control variate (CV) method of Kemna and Vorst (1990) is widely recommended in the literature and is regarded as the standard simulation method for Asian options. We call it classical CV method in the sequel. It can be used for basket options as well. There are few papers attempting to improve this control variate (e.g. Vázquez-Abad and Dufresne, 1998). However, these improvements are all moderate.

In this study, we develop a new variance reduction method for European basket and Asian options under the geometric Brownian motion (GBM) using formulas developed for approximation methods. Curran (1994) proposes an accurate approximation exploiting the dependency between the arithmetic and geometric average. We use this approximation to reduce the variance by suggesting a new control variate and combining it with conditional Monte Carlo and quadratic control variates. The new algorithm is fairly simple and reaches very large variance reduction.

In Section 2, we formulate and explain the basic principles of the naive simulation. Section 3 presents the classical and the new control variate methods. In Section 4, we introduce the conditional sampling, conditional Monte Carlo and quadratic control variates to improve the new control variate method. Section 5 reports our numerical results whereas Section 6 contains our conclusions.

2. Simulation of basket and Asian options

In this study, we deal with basket options and Asian options. The price of the former depends on the weighted arithmetic average of the prices of d different assets whereas the price of the latter depends on the prices of a single asset at d different time points. We consider the GBM model with constant volatilities σ_i for the asset price dynamics. We also assume a constant risk free interest rate r and constant continuous dividend yields δ_i. The weighted arithmetic average of the asset prices is given by

$$A = \sum_{i=1}^{d} w_i \Gamma_i,$$

where Γ_i's are the set of prices and w_i's are the weights of these prices. Here each Γ_i follows the log-normal distribution due to the GBM assumption. We also assume that each $w_i > 0$ and $\sum_{i=1}^{d} w_i = 1$.

For basket options, Γ_i denotes the price of the asset i at maturity T, d the number of assets, w_i the weight of the asset $i = 1, \ldots, d$. Let $S_i(t)$ denote the price of the asset i at time t. Then $\Gamma_i = S_i(T)$ and under GBM,

$$S_i(T) = S_i(0) \exp\left(\left(r - \delta_i - \sigma_i^2/2\right) T + \sigma_i W_i(T)\right), \quad i = 1, \ldots, d,$$

where $W_i(T), i = 1, \ldots, d$, are correlated standard Brownian motions with correlations ρ_{ij}.

For Asian options, Γ_i denotes the asset price at time t_i. That is, $\Gamma_i = S(t_i)$ for a single asset $S(t)$ following a GBM with risk free interest rate r, dividend yield δ and volatility σ. $0 = t_0 < t_1 < t_2 < \cdots < t_d = T$ are the control points in time, d is the number of control points and T is the maturity of the option. Also, each w_i equals to $1/d$. In this study, we consider the case of equidistant monitoring intervals, that is $t_i - t_{i-1} = \Delta t = T/d$, for $i = 1, 2, \ldots, d$. The proposed methods can easily be extended to the case of unequal intervals.

We restrict our attention to the pricing of call options with payoff function $P_A = (A - K)^+$ where K is the strike price, as the put–call parity automatically yields the price of the put option when the call option price is available, see Section 5.4.

The option price is given by the discounted risk neutral expectation of the payoff function: $e^{-rT} \mathrm{E}[P_A]$. To estimate the expectation by Monte Carlo simulation, we simulate n random payoffs. The sample mean of those payoffs gives us an estimate for the expectation. As $n \to \infty$, the estimator converges in distribution to the normal distribution. Thus we get an asymptotically valid confidence interval for the price estimate by using the quantile function of the standard normal distribution and the half width of the confidence interval is a probabilistic error bound for the price estimate.

For basket options, define R as $d \times d$ correlation matrix with entries $R_{ij} = \rho_{ij}$ and let L be the solution of $LL^T = R$ obtained by the Cholesky factorization (see Glasserman (2004, p. 73) for an algorithm to compute L). Then we get the following form used for the simulation

$$S_i(T) = S_i(0) \exp\left(\left(r - \delta_i - \sigma_i^2/2\right) T + \sigma_i \sqrt{T} \sum_{j=1}^{i} L_{ij} \xi_j\right), \quad i = 1, \ldots, d,$$

where $\xi_j, j = 1, \ldots, d$ are independent standard normal random variates. Note that the i-th element of the vector $L\xi$ can be written as $\sum_{j=1}^{i} L_{ij}\xi_j$ as L is lower triangular. This form requires $O(n\, d^2)$ computations for a simulation with sample size n. We present the details of the naive simulation as Algorithm 1.

For Asian options, the special structure of the correlation matrix of the prices at different time points yields a simple recursion to generate the asset price path that requires $O(n\, d)$ computations. We present the details in Algorithm 2.

3. Control variates

When using CVs the simulation output takes the form

$$Y_{CV} = Y - c(W - \mathrm{E}[W]),$$

where Y is the original (naive) simulation output and W is the control variate with (known) expectation $\mathrm{E}[W]$. Here, the optimal coefficient minimizing the variance is $c^* = \mathrm{Cov}(Y, W)/\mathrm{Var}(W)$ which corresponds to the least squares estimate of the slope of simple linear regression. It can be estimated by using a pilot run with a smaller sample size or by using the full sample of the simulation. The former approach leads to an unbiased estimate whereas the latter has a bias of order $O(1/n)$ which is negligible unless the sample size is small. When the optimal coefficient c^* is used, the variance reduction factor of the control variate method with respect to the naive simulation is $\mathrm{VRF} = 1/(1 - \rho_{YW}^2)$,