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Optimal dividend problem with a nonlinear regular-singular stochastic control

Mi Chen, Xiaofan Peng, Junyi Guo *

School of Mathematical Sciences, Nankai University, Tianjin 300071, China

HIGHLIGHTS

- A problem with a nonlinear regular-singular stochastic control is studied.
- Assume that the reinsurance premium is calculated according to the exponential premium principle.
- Both non-cheap and cheap reinsurance are investigated.
- Explicit expressions for the value function and the optimal strategies are obtained.

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ABSTRACT

In this paper, a problem with a nonlinear regular-singular stochastic control is studied for a big insurance portfolio. We assume that the reinsurance premium is calculated according to the exponential premium principle which makes the stochastic control problem nonlinear. Both non-cheap and cheap reinsurance are investigated. The objective of the insurer is to determine the optimal reinsurance and dividend policy so as to maximize the expected discounted dividends until ruin. Bounded dividend rates and unbounded dividend rates are considered. In both cases, explicit expressions for the value function and the corresponding optimal strategies are obtained. Finally, a numerical example is presented, which shows the impacts of risk aversion of the reinsurance company on the optimal value function and the retention level for reinsurance.

1. Introduction

The dividend problem for an insurance risk model was first proposed by De Finetti (1957) who proposed to look for the expected discounted sum of dividend payments until the time of ruin. Since then many research groups have tried to address the dividend problems under more general and more realistic model assumptions. The first papers which studied optimization of dividend distributions were Jeanblanc-Picqué and Shiryaev (1995) and Asmussen and Taksar (1997). Earlier works on risk control were given in, for example, Browne (1995). Subject to both dividend and reinsurance, the optimal dividend problem has been studied extensively. For example, see Højgaard and Taksar (1998a,b, 1999), Taksar and Zhou (1998), Asmussen et al. (2000) and Choulli et al. (2001, 2003). In addition to studies of dividend payout policies, problems associated with transaction costs and equity issuance, as mixed classical-impulse control problems, have attracted a lot of interest recently. Some examples include He and Liang (2008, 2009), Løkka and Zervos (2008), Bai et al. (2010), Meng and Siu (2011a,b) and Peng et al. (2012).

In most of the literature, premium is assumed to be calculated via the expected value principle for mathematical convenience. However, one can argue that two risks with the same mean may appear very different and the premium of them should also be different. For the optimal reinsurance problems under other premium principles, one can see Schmidli (2002), Young (1999), Kaluszka (2001, 2005) and Zhou and Yuen (2012). The exponential premium principle, which is the so-called zero utility principle, plays an important role in insurance mathematics and actuarial practice. It has many nice properties, including additivity with respect to independent risks. It is also widely used in mathematical finance to price various insurance products in the market. We refer the readers to

* Corresponding author. Tel.: +86 22 23506430.
E-mail addresses: chenmi0610@163.com (M. Chen), xfpengnk@yahoo.cn (X. Peng), jyguo@nankai.edu.cn (J. Guo).

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Young and Zariphopoulou (2002), Young (2003), Moore and Young (2003) and Musiela and Zariphopoulou (2004). More importantly, it is closely related to a new kind of nonlinear regular-singular stochastic control problem which is studied in this paper. A controlled nonlinear diffusion process in the context of excess-of-loss reinsurance has been widely studied in the past few years. For details, we refer the readers to Asmussen et al. (2000), Choulli et al. (2001), Bai et al. (2010), and Meng and Siu (2011a,b). Stochastic control problems where the drift of dynamics is quadratic in the regular control variable can be found in Guo et al. (2004) and Zhou and Yuen (2012).

The risk control in insurance takes on the form of reinsurance. There exist a variety of reinsurance forms. Among them, the most commonly used types of reinsurance include proportional, excess-of-loss and stop-loss. Under the exponential premium principle, the risk control becomes nonlinear which makes the problem more complicated than that under the expected value principle. So we choose proportional reinsurance in this paper for convenience.

The diffusion model is not directly appealing as a model for insurance purposes because clearly claims will cause jumps in the surplus process. But it can be seen as an approximation in a certain sense; such an approximation is suitable for big portfolios and it seems to be interesting from an optimal stochastic control point of view.

In this paper, we investigate the optimal dividend payments in the framework of diffusion model which is an approximation of the classical risk model with proportional reinsurance. We assume that the reinsurance premium is calculated according to the exponential premium principle which is closely related to a new kind of nonlinear regular-singular stochastic control problem. Zhou and Yuen (2012) studied the analogous problem for cheap reinsurance with the premium being calculated via the variance principle. They showed that there exists a common switch level for the optimal reinsurance and dividend policies instead of two levels under the expected value principle (see Højgaard and Taksar, 1999 or Løkka and Zervos, 2008). Not only cheap reinsurance but also non-cheap reinsurance are considered in this paper. Our objective is to maximize the expected discounted dividends until ruin. We obtain explicit expressions for the value function and the corresponding optimal strategies in two cases: one with unbounded dividend rates and the other with bounded dividend rates by positive upper bound M. In the case of unbounded dividend rates, we also show that there is only one common switch level for the optimal reinsurance and dividend policies, which is similar to those results under the variance principle rather than those under the expected value principle. But the case of bounded dividend rates is quite different.

The plan of this paper is as follows. In Section 2, we give a mathematical formulation of the diffusion model with proportional reinsurance and dividend payments under the exponential premium principle. In Section 3, we consider the problem in the case of no restriction on the rates of dividend payments. In Section 4, we study the problem with bounded rates of dividend payments. A numerical example is presented in Section 5. Finally, Section 6 concludes the paper.

2. The model

In this paper, all stochastic quantities are defined on a large enough complete probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbf{P})$; the filtration $\mathcal{F}_t$ represents the information available at time t and any decision made is based on this information.

Our results will be formulated within the controlled diffusion model. But we start with the classical Cramér–Lundberg model. In this model, the surplus process of the insurer without reinsurance and dividend payments is given by

$$U_t = x + ct - \sum_{i=1}^{N_t} Y_i,$$

where $x \geq 0$ is the initial surplus, $c > 0$ is the premium rate, $\{N(t), t \geq 0\}$ is a homogeneous Poisson process with intensity λ and $\{Y_i, i \geq 1\}$ is a sequence of positive i.i.d. random variables with common distribution $F(y)$. We denote by $\mu_1 = E(Y_i)$ its mean value and by $M_Y(r) = E(e^{rY_i})$ its moment generating function. We assume that the Cramér–Lundberg conditions hold, i.e., there exists $0 < r_\infty \leq \infty$, such that $M_Y(r) < \infty$ if $r < r_\infty$ and $\lim_{r\to r_\infty} M_Y(r) = +\infty$.

We assume that the insurer is allowed to reduce the risk by purchasing proportional reinsurance. The retention level for reinsurance is $\{b_t\}$, which means that the insurer pays b_tY of a claim occurring at time t, and the reinsurer pays $(1-b_t)Y$. Throughout this paper, we assume that the reinsurance premium is calculated according to the exponential premium principle. That is, for a risk U, the amount of premium $\pi_a(U)$ is determined by

$$\pi_a(U) = \frac{1}{a}\ln E(e^{aU}), \tag{2.1}$$

where the constant $a > 0$ measures the risk aversion of the reinsurance company and $\pi_a(U)$ is strictly increasing with respect to a; see Rolski et al. (1999) for details.

Similar to the cases with the expected value principle and the variance principle, the aggregate reinsurance premium under the exponential premium principle is also proportional to time t. Thus, the surplus process in the presence of proportional reinsurance (for a fixed retention level b) can be written as

$$U_t^b = x + \left[c - \frac{\lambda}{a}(M_Y(a(1-b)) - 1)\right]t - \sum_{i=1}^{N_t} bY_i. \tag{2.2}$$

It is well known that (2.2) can be approximated by a pure diffusion model X_t^b with the same drift and volatility. Specifically, if the retention level b changes with time and is stochastic, then the controlled reserve process X_t^b with the strategy b_t satisfies the following stochastic differential equation:

$$\begin{aligned} dX_t^b = {} & \left[c - \frac{\lambda}{a}(M_Y(a(1-b_t)) - 1) - \lambda b_t\mu_1\right]dt \\ & + \sqrt{\lambda\mu_2}\, b_t dW_t, \end{aligned} \tag{2.3}$$

with $X_0^b = x$, where $\{W_t, t \geq 0\}$ is a standard Brownian motion and μ_1, μ_2 are the first two moments of Y_i.

Remark 2.1. (i) Although (2.3) is a diffusion approximating process, the controlled variable b still depends on the distribution of claims (not only on the first two moments μ_1 and μ_2), which is different from those results with proportional reinsurance (see, for example, Højgaard and Taksar, 1998a,b, 1999, Taksar and Zhou, 1998, Choulli et al., 2003, Løkka and Zervos, 2008 and Zhou and Yuen, 2012).

(ii) For any fixed $b \in [0, 1]$, the drift coefficient of (2.3) is strictly decreasing with respect to $a > 0$, since $\varphi_1(a) = \frac{M_Y(a)-1}{a}$ is strictly increasing with respect to $a > 0$. In fact, we have

$$\varphi_1'(a) = \frac{aM_Y'(a) - (M_Y(a) - 1)}{a^2},$$
$$\varphi_2'(a) = aM_Y''(a) > 0 \quad \text{and} \quad \varphi_2(0) = 0,$$

where $\varphi_2(a) = aM_Y'(a) - (M_Y(a) - 1)$. Then it is obvious that $\varphi_1'(a) > 0$ for all $a > 0$.

In addition to purchasing proportional reinsurance, the insurance portfolio pays dividends to its shareholders by some dividend strategy. Denote by L_t the cumulative amount of dividends paid up to time t. Let $\alpha_t = (b_t, L_t)$ be the control process. When applying a