Insurance: Mathematics and Economics 51 (2012) 53–65

Contents lists available at SciVerse ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

# Alarm system for insurance companies: A strategy for capital allocation

S. Das [a], M. Kratz [b,*,1]

[a] IIM Bangalore, India

[b] ESSEC Business School Paris, France

ARTICLE INFO

*Article history:*
Received December 2010
Received in revised form
February 2012
Accepted 10 February 2012

*MSC:*
91B30
60K30

*Keywords:*
Alarm system
Capital accumulation function
Capital allocation
Quantitative risk management
Risk process
Ruin probability

ABSTRACT

One possible way of risk management for an insurance company is to develop an early and appropriate alarm system before the possible ruin. The ruin is defined through the status of the aggregate risk process, which in turn is determined by premium accumulation as well as claim settlement outgo for the insurance company. The main purpose of this work is to design an effective alarm system, i.e. to define alarm times and to recommend augmentation of capital of suitable magnitude at those points to reduce the chance of ruin. To draw a fair measure of effectiveness of alarm system, comparison is drawn between an alarm system, with capital being added at the sound of every alarm, and the corresponding system without any alarm, but an equivalently higher initial capital. Analytical results are obtained in general setup and this is backed up by simulated performances with various types of loss severity distributions. This provides a strategy for suitably spreading out the capital and yet addressing survivability concerns at factory level.



## 1. Introduction and overview

This work develops an early and appropriate alarm system for an insurance institution before its possible ruin based on pattern of premium collection and demands for claim settlement. While keeping a very high initial capital may avoid ruin for the insurance company, it is neither desired by most companies because of obvious investment concerns, nor is it feasible at times. An effective alarm system opens the door to an alternate strategy based on ruin theory by opting for less initial capital and topping it up when really necessary.

Alarm systems have been developed in different contexts in the literature (viz. Guillou et al., 2010, Lindgren, 1980 and Monteiro et al., 2008 and references therein), while capital reserving or capital allocation have been addressed in many articles (viz. Besson et al., 2009, Kaishev et al., 2007, and references therein). In particular, Kaishev et al. (2007), showed numerically that two capital accumulation functions, one linear and the other piecewise linear with one jump at some instances, would lead to equal chances of survival and also equal accumulated risk capital at the end of the considered time interval. The approach in the present work, with the introduction of a new alarm system, is fundamentally different even though the broad concern is similar, i.e. to reduce the initial capital without compromising on the survival probability.

The basic idea behind our proposed notion of alarm is as follows. Alarm is sounded at a juncture when the probability of ruin (in the absence of any intervention) within a specified future time period is *high*. While few variations in defining the alarm time have been explored in Das and Kratz (2010), we find it more appropriate when the above probability is set in terms of conditional probability of ruin given survival up to the alarm time. In addition, we require that the probability of non-ruin before the alarm should be sufficiently high. An alarm system consisting of a sequence of alarms is defined following a natural extension of the single alarm and with the addition of capital at the sound of each alarm. This system constitutes an alternate strategy for having to put up an excessive initial capital to avoid ruin.

Note that this strategy does not interfere with the Value-at-Risk approach (or any tail approach) applied by insurance companies as mandated by the Solvency regulation. It just means that the capital may also be adjusted on a regular basis (e.g. every quarter) for the risk adjusted capital to be higher than the capital required by Solvency.

For a fair evaluation of the effectiveness of our strategy, the proposed alarm system is pitted against a default no-alarm system equipped with equivalent higher initial capital. We compare the

* Corresponding author.
*E-mail address:* kratz@essec.edu (M. Kratz).

1 Member of MAP5, UMR8145, Univ. Paris Descartes.

0167-6687/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.insmatheco.2012.02.009

survival probabilities under the alternatives. In the long run, the alarm system is expected to perform better in terms of higher survival probability over finite horizon, as is indeed confirmed by our study. Consequently, the alarm system may be preferred even if the chance of survival under this is marginally worse in immediate future or very short horizon. With that being the objective, we focus on analytical as well as numerical evaluation of the comparative survival probabilities under the two systems.

To illustrate the formulation and methodology of our alarm system, we consider a model as simple as possible, by taking a linear accumulation model with i.i.d. claims. However our method can be adapted to any structural changes in the model, arising from realistic considerations like Solvency requirements, dependent claims, changes of claim reserves, investment losses and incomes. (For instance, the adjustment of capital as mandated by Solvency rules might be easily accommodated in the setup using a stepwise linear accumulation function.) While the alarm times would change with these modifications, the main principles would remain the same. So we avoid complex model not to obscure the basic idea unnecessarily.

Note also that if the stochastic nature of the risk process is completely known, as is assumed in this work, the alarm times are fixed known parameters, depending on various parameters of the underlying risk process. In practice, the proposed mechanism may be embedded into an adaptive scheme, where additional information regarding the risk process in terms of claims would be recursively/progressively utilized to lead to a suitable random alarm system that draws on empirical information on claims.

The paper is organized as follows. Section 2.1 introduces the basic notation and framework of the work. In Section 2.2, we give the formal definition of alarm time, choose few examples to cover the different types of severity distributions, discrete to continuous, as well as light vs. heavy tail, and study the role of various parameters in the definition of alarm times. Formalization of multiple alarms leading to an alarm system is taken up in Section 2.3. The next section, Section 3, develops a strategy to alleviate initial capital using alarm systems. The effectiveness of alarm systems and comparison across the different options including that of not adopting any alarm system is discussed here. The numerical demonstrations are provided in Section 3.1. General analytical bounds are derived in Section 3.2, providing directions of adaptability of the alarm system in specific real circumstances.

## 2. Alarm system based on probability of impending ruin

### 2.1. Framework

To present our approach, we consider the simple ruin theory model, namely the Cramér Lundberg model, although much of the analysis of this paper can be carried over in a straightforward way to more general Lévy processes and premium rates. Most of the definitions and results will be given in terms of ruin probability since results on distributions of ruin time can be found in the existing literature; see some key references in this decade, e.g. Asmussen and Albrecher (2010), Dickson (2005), Embrechts et al. (2001), Ignatov and Kaishev (2004, 2006), Kaishev and Dimitrova (2006) and Mikosch (2004); in particular the finite time survival probability is expressed for continuous and discrete claims, respectively in Ignatov and Kaishev (2004, 2006). From broad considerations, ruin time distributions depend on whether the claims distributions are continuous or discrete, but also on the characterization of the claims tail distributions; our choice of examples will attempt to reflect that.

By default, we assume that the claim amounts (severity) $X_i$'s are i.i.d. with distribution function $F$ and mean $\mu$, with i.i.d. inter-arrival times $T_k - T_{k-1}$ exponentially distributed and independent of the $X_i$'s. We will consider various different claim distributions $F$. Set $T_0 = 0$. The aggregated claims $(S_s)_{s\geq 0}$ are defined by $S_s = \sum_{i=1}^{N_s} X_i$, where $N_s = \sup\{k \geq 1 : T_k \leq s\}$ is a homogeneous Poisson process with intensity $\lambda > 0$.

Consider the risk (or surplus) process $(V_s^{u_s})_{s\geq 0}$ defined by

$$V_s^{u_s} = u_s + p_s - S_s = u_s - R_s, \tag{1}$$

where $u_s$ denotes the capital function at time $s$, the premium rate is linear, viz. $p_s = cs$, and the net outgo (without taking the capital into account, i.e. aggregate claims less premium collected) is given by $R_s = S_s - p_s$. Note that while $(R_s)$ is a stochastic process (a compound Poisson process in our setting), the capital process $(u_s)$ is non-random and at the discretion of the company. One of the key objectives of this work may be restated as the determination of $u_s$ given the knowledge of parameters of $R_s$.

The ruin time of such a risk process is then formally defined as:

$$T = \inf\{s > 0 : V_s^{u_s} < 0\} = \inf\{s > 0 : R_s > u_s\}, \tag{2}$$

with $T = \infty$ if there is no ruin. Note that while in practice one may wish to define ruin as the first time instance when $V_s^{u_s}$ goes below a level $L$ (other than 0), it would take only a trivial adjustment in the approach adopted here. Consequently, in this work, we stick to $L = 0$.

Of special interest is starting with an initial capital $u_0 = u$ and not making any further addition, i.e. $u_s = u$ for all $s \geq 0$; it will be our benchmark or starting framework. The ruin time in such a case will be denoted by $T(u)$, i.e.

$$T(u) = \inf\{s > 0 : V_s^u < 0\} = \inf\{s > 0 : R_s > u\}. \tag{3}$$

Note that while flexibility in the choice of the initial capital $u$ is an integral part of this work, in a given instance we are concerned with a fixed value for $u$ and will not be interested in the asymptotic behavior as $u \to \infty$, unlike most related literature.

At times, we are interested in the behavior of the surplus process only after a given time $a \geq 0$, with constant capital function beyond this time, $u_s = u$, $\forall s \geq a$; in such a case, let $T(a, u)$ denote the ruin time defined by

$$T(a, u) = \inf\{s > a : V_s^u < 0\} = \inf\{s > a : R_s > u\}.$$

In particular, $T(u) \equiv T(0, u)$.

The infinite horizon ruin probability with capital $u$ at time $a$ is denoted by:

$$\psi_a(u) := P[T(a, u) < \infty] = P[\inf_{s>a} V_s^u < 0] = P[\sup_{s>a} R_s > u].$$

The corresponding finite horizon ruin probability, which is the distribution function of the r.v. $T(a, u)$, is given by

$$\psi_a(u, t) := P[T(a, u) \leq t] = P[\sup_{a<s\leq t} R_s > u].$$

To simplify the notation, we set

$$\psi_0(u) = \psi(u); \qquad \psi_0(u, t) = \psi(u, t); \quad \text{and}$$
$$\bar{\psi}_a(u, t) = 1 - \psi_a(u, t).$$

Let us introduce the conditional ruin probabilities in infinite and finite times given some event $B$:

$$\begin{aligned} \psi_a(u \mid B) &:= P[T(a, u) < \infty \mid B] \\ &= P[\inf_{s>a} V_s^u < 0 \mid B] \\ &= P[\sup_{s>a} R_s > u \mid B], \end{aligned}$$

and

$$\psi_a(u, t \mid B) \quad := P(T(a, u) \in (0, t] \mid B) = P[\sup_{a\leq s\leq t} R_s > u \mid B].$$