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Claims development result in the paid-incurred chain reserving method

Sebastian Happ [a,*], Michael Merz [a], Mario V. Wüthrich [b]

[a] *University of Hamburg, Faculty of Economics, 20146 Hamburg, Germany*
[b] *ETH Zurich, RiskLab, Department of Mathematics, 8092 Zurich, Switzerland*

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ABSTRACT

We present the one-year claims development result (CDR) in the paid-incurred chain (PIC) reserving model. The PIC reserving model presented in Merz and Wüthrich (2010) is a Bayesian stochastic claims reserving model that considers simultaneously claims payments and incurred losses information and allows for deriving the full predictive distribution of the outstanding loss liabilities. In this model we study the conditional mean square error of prediction (MSEP) for the one-year CDR uncertainty, which is the crucial uncertainty view under Solvency II.

1. Introduction

A non-life insurance company needs to hold sufficient claims reserves (provisions) on its balance sheet in order to meet the outstanding loss liabilities. Therefore, a main task of the actuary in non-life insurance is to predict ultimate loss ratios and outstanding loss liabilities. For these predictions he often has different sources of information and the major difficulty is to combine these information channels appropriately.

In the present paper we combine claims paid data and claims incurred data (case estimates for reported claims) to get a unified prediction for the outstanding loss liabilities. A well known method to combine claims paid data and claims incurred data for claims reserving is the Munich chain ladder (MCL) method introduced in Quarg and Mack (2004). However, to the best of our knowledge, there is no way to quantify the prediction uncertainty within the MCL method. Another approach was presented in Dahms (2008). Dahms (2008) extended the complementary loss ratio (CLR) method for deriving unified predictions based on claims paid data and claims incurred data simultaneously. Unlike the MCL method, the CLR method allows for the derivation of a mean square error of prediction (MSEP) estimate. A recent new approach is the paid–incurred (PIC) reserving method introduced in Posthuma et al. (2008) and Merz and Wüthrich (2010). The PIC method was defined in a Bayesian framework and therefore allows for the derivation of the full predictive distribution for the outstanding loss liabilities. This means that within the Bayesian PIC model one is not only able to calculate the MSEP but one can also calculate any other risk measure, like Value-at-Risk or expected shortfall for the prediction uncertainty.

Under the new solvency regulations, such as Solvency II, the so-called one-year claims development result (CDR) is of central interest because it corresponds to a profit and loss statement position that directly influences the financial strength of an insurance company. The one-year CDR is defined as the difference between the prediction of the outstanding loss liabilities today and in one year's time (cf. Merz and Wüthrich, 2008). This means that the one-year CDR measures the change in the expected outstanding loss liabilities over a one-year time horizon. Due to Solvency II, this one-year view has attracted a lot of attention in recent research. For references, we refer to Ohlsson and Lauzeningks (2009), Merz and Wüthrich (2008) and Bühlmann et al. (2009). Dahms et al. (2009) analyze the one-year CDR in the framework of the CLR method, which is probably the first one-year CDR uncertainty analysis for combined claims paid and claims incurred data.

In the present paper we revisit the PIC method within this one-year solvency framework. This means that we consider the one-year CDR for the PIC reserving method. We are able to calculate

* Corresponding author. Tel.: +49 0 40 428388372.
E-mail address: sebastian.happ@wiso.uni-hamburg.de (S. Happ).

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the conditional MSEP for the one-year CDR and we can also derive the full predictive distribution of the one-year CDR via Monte-Carlo simulations.

Organization of the paper: In Section 2 we recapitulate the assumptions of the PIC model. The definition of the one-year CDR is given in Section 3. We then derive the best estimate of the ultimate claim, based on the paid and incurred data in one year, see Section 4. In Section 5.1 we split this best estimate in an appropriate way and derive the conditional MSEP of the one-year CDR for single accident years. In Section 5.2 we proceed with the conditional MSEP for aggregated accident years which provides the overall one-year CDR uncertainty. Finally, in Section 6 we present an example and compare it to the results derived in Dahms et al. (2009) for the CLR method. Additionally, we provide the full predictive distribution of the one-year CDR via Monte-Carlo simulations. All proofs are provided in the Appendix.

2. Notation and model assumptions

The PIC reserving model combines two channels of information: (i) claims payments, which correspond to the payments for reported claims; (ii) incurred losses, which refer to the reported claim amounts. Claims payments and incurred losses data are usually aggregated in so-called claims development triangles:

In the following, we denote accident years by $i \in \{0, \dots, J\}$ and development years by $j \in \{0, \dots, J\}$. Cumulative payments in accident year i after j development years are denoted by $P_{i,j}$ and the corresponding incurred losses by $I_{i,j}$. We assume that all claims are settled and closed after development year J, i.e. $P_{i,J} = I_{i,J}$ holds with probability 1 for all $i \in \{0, \dots, J\}$. After accounting year $t = J$ we have observations in the paid and incurred triangles given by (see Fig. 1)

$$\mathcal{D}_J = \{P_{i,j}, I_{i,j};\ 0 \le i \le J, 0 \le j \le J, 0 \le i + j \le J\},$$

and after accounting year $t = J + 1$ we have observations in the paid and incurred trapezoids given by (see Fig. 2)

$$\mathcal{D}_{J+1} = \{P_{i,j}, I_{i,j};\ 0 \le i \le J, 0 \le j \le J, 0 \le i + j \le J + 1\}.$$

This means the update of information $\mathcal{D}_J \mapsto \mathcal{D}_{J+1}$ adds a new diagonal to the observations. Our goal is to predict the ultimate losses $P_{i,J} = I_{i,J}$, $i = 1, \dots, J$, based on the information $\mathcal{D}_J$ and $\mathcal{D}_{J+1}$, respectively.

We define the log-normal PIC model, which combines both cumulative payments and incurred losses information:

Model Assumptions 2.1 (*Log-Normal PIC Model*).

- Conditionally, given the parameter $\Theta = (\Phi_0; \Phi_1, \Psi_1, \Phi_2, \Psi_2, \dots, \Phi_J, \Psi_J)'$, we assume:
  - the random vectors $\Xi_i = (\xi_{i,0}; \xi_{i,1}, \zeta_{i,1}, \xi_{i,2}, \zeta_{i,2}, \dots, \xi_{i,J}, \zeta_{i,J})'$ are i.i.d. with multivariate Gaussian distribution
    $\Xi_i \sim \mathcal{N}(\Theta, V)$ for $i \in \{0, 1, \dots, J\}$,
    with positive definite covariance matrix V and individual development factors
    $$\xi_{i,j} = \log \frac{P_{i,j}}{P_{i,j-1}} \quad \text{and} \quad \zeta_{i,l} = \log \frac{I_{i,l}}{I_{i,l-1}},$$
    for $j \in \{0, 1, \dots, J\}$ and $l \in \{1, 2, \dots, J\}$, where we have set $P_{i,-1} = 1$;
  - $P_{i,J} = I_{i,J}$, $\mathbb{P}$-a.s., for all $i = 0, 1, \dots, J$.
- The components of Θ are independent with prior distributions
  $$\Phi_j \sim \mathcal{N}(\phi_j, s_j^2) \quad \text{for } j \in \{0, \dots, J\} \quad \text{and}$$
  $$\Psi_l \sim \mathcal{N}(\psi_l, t_l^2) \quad \text{for } l \in \{1, \dots, J\},$$
  with prior parameters $\phi_j, \psi_l \in \mathbb{R}$ and $s_j^2 > 0$, $t_l^2 > 0$.

Remarks.

- Note that in Model Assumptions 2.1 we can choose any arbitrary positive definite covariance matrix V. This even allows for modelling dependence structures between claims payments ratios and incurred losses ratios.
- Expert opinion should be included to structure the covariance matrix V. For a more detailed discussion on this topic and suitable choices for V we refer to Happ and Wüthrich (2011). However, the problem of finding statistically optimal estimators should be subject to more statistical research.
- We define prior distributions for the components of the mean vector Θ and assume V to be a given covariance matrix. This Bayesian approach guarantees closed form results. If we also put a prior on V we have to calculate the posterior distribution of V using Markov-Chain-Monte-Carlo (MCMC) methods (see Merz and Wüthrich, 2010).

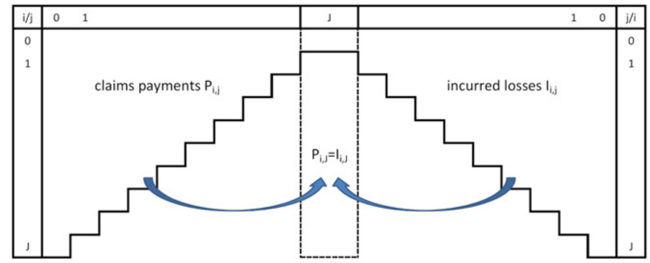

Fig. 1. Cumulative claims payments $P_{i,j}$ and incurred losses $I_{i,j}$ observed after accounting year $t = J$ both leading to the ultimate loss $P_{i,J} = I_{i,J}$.

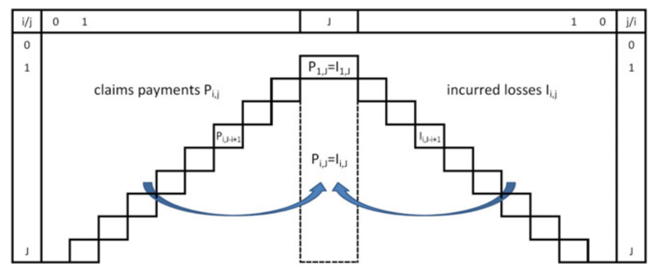

Fig. 2. Cumulative claims payments $P_{i,j}$ and incurred losses $I_{i,j}$ observed after accounting year $t = J + 1$ both leading to the ultimate loss $P_{i,J} = I_{i,J}$.

3. One-year claims development result

In this paper we consider the short term (one-year) run-off risk described in Merz and Wüthrich (2008). This means, we study the uncertainty in the one-year CDR for accounting year $J + 1$ given by

$$\begin{aligned}\mathrm{CDR}_i &= \mathrm{CDR}_i(J+1) \\ &= \mathbb{E}\left[P_{i,J}|\mathcal{D}_J\right] - \mathbb{E}\left[P_{i,J}|\mathcal{D}_{J+1}\right], \quad i = 1, \dots, J,\end{aligned}$$

between the best estimates for the ultimate claim $P_{i,J}$ at times J and $J + 1$.

The one-year CDR in accounting year $J + 1$ measures the change in the prediction by updating the information from $\mathcal{D}_J$ to $\mathcal{D}_{J+1}$. With the tower property of the conditional expectation we obtain for the expected one-year CDR for accident year i, viewed from time J,

$$\mathbb{E}\left[\mathrm{CDR}_i|\mathcal{D}_J\right] = 0,$$

which is the martingale property of successive predictions. This justifies the fact that, in the budget statement, the one-year CDR is usually predicted by 0 at time J. In the following we study the uncertainty in this prediction by means of the conditional MSEP, given the observations $\mathcal{D}_J$. In other words we calculate, see Wüthrich and Merz (2008, Section 3.1),

$$\begin{aligned}\mathrm{msep}_{\mathrm{CDR}_i|\mathcal{D}_J}(0) &= \mathbb{E}\left[(\mathrm{CDR}_i - 0)^2\,|\mathcal{D}_J\right] = \mathrm{Var}\left(\mathrm{CDR}_i|\mathcal{D}_J\right) \\ &= \mathrm{Var}\left(\mathbb{E}\left[P_{i,J}|\mathcal{D}_{J+1}\right]|\mathcal{D}_J\right). \qquad (1)\end{aligned}$$