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Optimal asset allocation for DC pension plans under inflation

Nan-wei Han [a], Mao-wei Hung [b,*]

[a] Department of Banking and Finance, College of Finance, Takming University of Science and Technology, No. 56, Section 1, Huanshan Road, Taipei, Taiwan
[b] Department of International Business, College of Management, National Taiwan University, No. 1, Section 4, Roosevelt Road, Taipei, Taiwan

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ABSTRACT

In this paper, the stochastic dynamic programming approach is used to investigate the optimal asset allocation for a defined-contribution pension plan with downside protection under stochastic inflation. The plan participant invests the fund wealth and the stochastic interim contribution flows into the financial market. The nominal interest rate model is described by the Cox–Ingersoll–Ross (Cox et al., 1985) dynamics. To cope with the inflation risk, the inflation indexed bond is included in the asset menu. The retired individuals receive an annuity that is indexed by inflation and a downside protection on the amount of this annuity is considered. The closed-form solution is derived under the CRRA utility function. Finally, a numerical application is presented to characterize the dynamic behavior of the optimal investment strategy.

1. Introduction

In the design of pension schemes, two main types of plan are usually adopted — the defined-benefit (DB) plan and the defined-contribution (DC) plan. In DB plans, the benefits are defined in advance and contributions are set and subsequently adjusted to keep the fund in balance. In DC plans, only the contributions are defined and the financial risk is shifted from the sponsor to the contributor. Historically, the DB scheme was definitely preferred by the employees because of the simplicity and security of the payout pattern it offers as base coverage. However, in recent years, DC plans have become popular in the pension market due to the demographic evolution and the development of the equity markets.

Since the retirement benefits of DC pension plans depend on the returns on fund's portfolios, the asset allocation decisions are crucial to DC pension management. Some earlier research has solved the optimal asset allocation rules for DC pension plans under constant interest rates and neglected the effect of inflation (see, e.g. Browne, 1999; Cairns et al., 2006; Gerrard et al., 2004). However, as the investment of a pension plan involves quite a long period, usually 20–40 years, it seems implausible to assume a constant level of interest rates and ignore the inflation in the long run.

In this research, we propose a continuous-time model for the optimal DC pension plan management with stochastic interest rates and inflation. The framework of this research could be characterized as follows. First, the Cox–Ingersoll–Ross (CIR) process is employed to model the dynamics of nominal interest rates. The CIR process does not permit negative interest rates, which is a more desirable feature when modeling the nominal interest rates. Next, the instantaneous inflation is assumed to follow a diffusion process. The inflation rates are partially correlated to the interest rates and the stock returns. As a result, the inflation risk cannot be perfectly hedged by any mix of nominal bonds and stocks. In order to complete the financial market and hedge against the inflation risk more efficiently, the inflation-indexed bonds are included in the financial market.

To prevent shortfalls on final benefits and make the DC pension scheme more attractive to the public, various types of guarantee on the terminal benefits can be included in DC pension plans (see, e.g. Boulier et al., 2001; Cairns et al., 2006; Deelstra et al., 2003; Pennacchi, 1999; Vigna and Haberman, 2001). In this research, a minimum guarantee is put on the amount of an inflation-indexed

* Corresponding author. Tel.: +886 233664988; fax: +886 223690833.
E-mail addresses: nanweihan@takming.edu.tw (N.-w. Han), Hung@management.ntu.edu.tw (M.-w. Hung).

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annuity at the retirement date. This helps the annuitants to hedge against the inflation risk during the annuitization phase. Since the value of the indexed annuity could be viewed as an indexed coupon bond, the value of the guarantee is therefore stochastic and contingent to the interest rate and the price level at the retirement date.

This study extends the previous works of Boulier et al. (2001), Deelstra et al. (2003), and Battocchio and Menoncin (2004). Boulier et al. (2001) and Deelstra et al. (2003) considered a minimum guarantee included in the DC pension plan, while the inflation risk was ignored in both works. Battocchio and Menoncin (2004) considered the effect of inflation risk on the optimal DC pension management but the minimum guarantee was absent in their model. Furthermore, in Battocchio and Menoncin (2004), the inflation-indexed bond was not included in the economy. As a result, there was no real riskfree asset in the financial market and the individuals cannot adjust their risk exposure according to their risk aversion. In this paper, we show that the inclusion of indexed bond in the economy helps us to hedge against the inflation risk perfectly during the accumulation phase and also makes it possible to replicate the required guarantee efficiently.

The objective of the pension plan is assumed to maximize the participant's expected utility defined over the real fund wealth in excess of the minimum guarantee at the retirement date. The optimization problem is solved by the stochastic dynamic programming approach, which was firstly used in the pioneering work of Merton (1969). Battocchio and Menoncin (2004) and Gao (2008) applied the stochastic dynamic programming approach to solve the optimal investment rule for the DC pension plan without downside protection. Our research shows that by using the technique of change of variables, the optimal asset allocation problem for a protected DC pension plan — which is initially non-self-financing and constrained by the minimum guarantee; could be transformed into a self-financing and unconstrained one. In this case, the standard stochastic dynamic programming approach can be easily applied to find the explicit solution of the optimal investment rules in our model.

A numerical exercise is carried out to show the age-dependent dynamics of the optimal portfolio weights. The results show that the portfolio weights on stocks and nominal bonds are initially high and then decrease over time. On the other hand, the portfolio weights on inflation-indexed bonds and cash are initially low and then increase over time. In contrast with the findings of the previous works which did not include the indexed bond in the economy, the inflation-indexed bond dominates the pension portfolio for the participant who is more risk averse and concerned about the shortfall on retirement benefits. This reflects the fact that the indexed bond, which delivers certain real payoffs when it matures is the only riskfree asset (in real terms) in the long run. The numerical example also shows that the aggressive investors would hold short positions in indexed bonds to earn the inflation risk premium during the accumulation phase.

2. Background

As we know, the goal of a pension plan is to finance the contributor's living after retirement. Since the terminal fund wealth at the retirement date depends on the performance of the fund portfolio during the accumulation phase, a well-managed DC pension fund should make every effort to earn revenues by investing in the financial markets and seek to provide the participants with a downside protection on the final benefits. This research aims to investigate the optimal asset allocation rule for a protected DC plan during the accumulation phase with risks of interest rates and inflation. The downside protection is designed as a minimum guarantee on the retirement annuity which is inflation-indexed.

2.1. Interest rate risk

As the investment decision for a pension plan involves quite a long period, it is reasonable to assume that the interest rates are stochastic in the long run. Theoretically, if the interest rates are assumed to be constant, there will be no difference between bonds and cash accounts. In the literature on optimal asset allocation, the Ornstein–Uhlenbeck process of Vasicek (1977) is the most popular model to describe the dynamics of interest rates. The interest rate model of Vasicek (1977) was used in Battocchio and Menoncin (2004) and many earlier works (see, e.g. Sørensen, 1999; Brennan and Xia, 2000; Boulier et al., 2001). The advantage of Vasicek's model is that the interest rates are normally distributed and, as a result, the bond price and the optimal portfolio strategy are easy to solve. However, one debatable problem of the Vasicek's model is that it allows the interest rates to be negative, an unfavorable property when modeling for the dynamics of nominal interest rates.

In our research, the dynamics of the nominal interest rate are modeled by the square-root CIR process suggested by Cox et al. (1985). In the CIR framework, the interest rates are non-central chi-square distributed and the solutions for bond price and optimal portfolio strategy are more intractable. However, the most important advantage of this model is that it does not permit negative interest rates. This is a more realistic assumption, especially in modeling the nominal interest rates. In the works of Deelstra et al. (2003) and Gao (2008), the dynamics of interest rates were described by a general affine process proposed by Duffie and Kan (1996). Both the Vasicek and the CIR processes could be viewed as special cases of the general affine process. However, the numerical analyses in both works were carried out for the CIR case.

2.2. Inflation risk

In addition to the risk of interest rate, the investment period is too long for a DC pension plan to neglect the effect of inflation. The accumulated inflation in the long run could severely damage the real payoff of the pension plan.

The asset allocation problem incorporating inflation risk for individual investors has been surveyed in some earlier papers (see, e.g. Brennan and Xia, 2002; Campbell and Viceira, 2001; Munk et al., 2004). The optimal asset allocation rule under inflation for a DC pension fund was investigated by Battocchio and Menoncin (2004). The interest rates, contribution flows, and inflation rates were assumed to be stochastic in the model. The goal of the pension plan is to maximize the expected utility over the fund's real wealth at the retirement date. They argued that the fund manager could hedge against the inflation risk through a portfolio dominated by cash. The cash, which is an instantaneously riskfree asset in the nominal economy, bears inflation risk in real terms. Since there were three kinds of risks existing in the economy, i.e. the interest rate risk, the equity risk, and the inflation risk, these three risky assets were able to complete the financial market in terms of real returns. We note that, since the cash was changed into a risky asset in real terms, there did not exist a single real risk-free asset to be traded and the riskfree portfolio must deliver zero real returns. This implies that the investors cannot arbitrarily adjust their risk exposure according to their risk aversion.

In order to improve the market, we consider an additional asset class, the inflation-indexed bond, in the financial economy. The indexed bond is defined as an instrument that delivers a defined payoff indexed by the inflation at its maturity. It serves as the real riskfree asset for a long-horizon investment. Furthermore, the returns on the indexed bond would be highly correlated to the inflation and this helps the investor of the indexed bond to hedge against the inflation risk. Campbell and Viceira (2001) showed