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Forecasting mortality for small populations by mixing mortality data

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HIGHLIGHTS

- We address the projection of mortality when data show major random fluctuations.
- The mortality of the small group is 'replicated' by mixing data of other groups.
- The procedure is successful in face of major erratic movements in mortality data.

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ABSTRACT

In this paper we address the problem of projecting mortality when data are severely affected by random fluctuations, due in particular to a small sample size, or when data are scanty. Such situations may emerge when dealing with small populations, such as small countries (possibly previously part of a larger country), a specific geographic area of a (large) country, a life annuity portfolio or a pension fund, or when the investigation is restricted to the oldest ages. The critical issues arising from the volatility of data due to the small sample size (especially at the highest ages) may be made worse by missing records; this is the case, for example, of a small country previously part of a larger country, or a specific geographic area of a country, given that in some periods mortality data could have been collected just at an aggregate level.

We suggest to 'replicate' the mortality of the small population by mixing appropriately the mortality data obtained from other populations. We design a two-step procedure. First, we obtain the average mortality of 'neighboring' populations. Three alternative approaches are tested for the assessment of the average mortality; conversely, the identification and the weight of the neighboring populations are obtained through (standard) optimization techniques. Then, following a sort of credibility approach, we mix the original mortality data of the small population with the average mortality of the neighboring populations.

In principle, the approach described in the paper could be adopted for any population, whatever is its size, aiming at improving mortality projections through information collected from other groups. Through backtesting, we show that the procedure we suggest is convenient for small populations, but not necessarily for large populations, nor for populations not showing noticeable erratic effects in data. This finding can be explained as follows: while the replication of the original data implies the increase of the size of the sample, it also involves a smoothing of data, with a possible loss of specific information relating to the group referred to. In the case of small populations showing major erratic movements in mortality data, the advantages gained from the larger sample size overcome the disadvantages of the smoothing effect.

1. Introduction

There is a growing interest in mortality projections for small populations, such as small countries (possibly previously part of a larger country), a specific geographic area of a (large) country, a life annuity portfolio or a pension fund, or when restricting the investigation to the oldest ages. Such an interest is motivated by two main needs: to improve projection accuracy, and to assess the basis risk when hedging longevity risk through longevity securities.

With regard to projection accuracy we note that, due to the small sample size, data for small populations may be subject to major random fluctuations; further, some data could be missing. The latter problem is quite common when dealing with a small country previously part of a larger country, or when dealing with

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Table 1
Basket of countries.

Code	Country	Code	Country	Code	Country	Code	Country
AT	Austria	FI	Finland	LV	Latvia	PT	Portugal
BE	Belgium	FR	France	LT	Lithuania	SCO	Scotland
BG	Bulgaria	DE	Germany	LU	Luxembourg	SK	Slovakia
CZ	Czech Republic	HU	Hungary	NL	Netherlands	SI	Slovenia
DK	Denmark	IS	Iceland	NIE	Northern Ireland	ES	Spain
ENG	England & Wales	IE	Ireland	NO	Norway	SE	Sweden
EE	Estonia	IT	Italy	PL	Poland	CH	Switzerland

Table 2
Data volatility and size of the countries in the basket.

Country	$\bar{\mathbb{CV}}[\tilde{m}_{x,t}^{[k]}]$		$\frac{\bar{E}^{[k]}}{\sum_{k=0}^{n}\bar{E}^{[k]}}$	
	Males	Females	Males	Females
AT	5.634%	4.190%	1.715%	1.920%
BE	4.490%	3.691%	2.435%	2.370%
BG	5.116%	4.418%	2.043%	1.803%
CZ	5.993%	4.063%	2.099%	2.196%
DK	5.617%	4.798%	1.279%	1.170%
ENG	1.888%	1.391%	14.115%	13.565%
EE	14.800%	8.994%	0.253%	0.349%
FI	8.323%	5.926%	1.035%	1.121%
FR	1.929%	1.612%	12.822%	12.796%
DE	1.887%	1.439%	15.174%	16.556%
HU	5.278%	3.806%	2.214%	2.367%
IS	30.281%	29.302%	0.049%	0.040%
IE	8.449%	6.865%	0.700%	0.600%
IT	1.899%	1.622%	14.345%	13.789%
LV	9.936%	6.600%	0.437%	0.604%
LT	7.557%	6.000%	0.613%	0.750%
LU	25.484%	20.884%	0.086%	0.086%
NL	3.720%	3.293%	3.167%	2.980%
NIE	12.340%	9.330%	0.329%	0.320%
NO	6.330%	5.679%	1.055%	0.967%
PL	3.152%	2.291%	6.320%	6.796%
PT	5.271%	4.008%	2.284%	2.190%
SCO	6.639%	4.578%	1.196%	1.208%
SK	8.740%	6.504%	0.884%	0.900%
SI	14.253%	10.191%	0.361%	0.401%
ES	2.431%	2.101%	9.004%	8.493%
SE	4.304%	3.845%	2.405%	2.135%
CH	5.646%	4.811%	1.580%	1.530%

Table 4
Parameter calibration and backtesting, method (Δm).

	$\text{MSE}^{[0]}$		$\bar{z}^{(\Delta m)}$		$\frac{\text{MSE}^{[R0]}}{\text{MSE}^{[0]}}$	
	Males	Females	Males	Females	Males	Females
AT	0.00118	0.00037	0.16305	0.18652	71.61%	119.40%
BE	0.00142	0.00055	0.20194	0.20645	67.83%	53.25%
BG	0.01282	0.00526	0.17314	0.15120	26.19%	54.11%
CZ	0.00439	0.00038	0.21527	0.37682	40.20%	120.18%
DK	0.00081	0.00023	0.24608	0.13066	95.74%	73.15%
ENG	0.00021	0.00011	0.76830	0.81640	87.76%	75.09%
EE	0.01886	0.01121	0.14872	0.14495	59.94%	8.79%
FI	0.00326	0.00064	0.10578	0.11435	47.76%	84.91%
FR	0.00021	0.00009	0.58306	0.65259	62.78%	95.00%
DE	0.00091	0.00039	0.61949	0.63962	56.84%	60.45%
HU	0.00243	0.00061	0.21494	0.22083	53.68%	71.40%
IS	0.00959	0.00452	0.03787	0.00889	117.31%	53.01%
IE	0.00366	0.00045	0.06586	0.05272	50.56%	106.68%
IT	0.00050	0.00015	0.66494	0.70404	47.54%	47.07%
LV	0.02605	0.00118	0.26265	0.40933	22.67%	54.26%
LT	0.00738	0.00135	0.29243	0.30072	101.59%	27.34%
LU	0.07696	0.01095	0.03233	0.03433	21.76%	34.63%
NL	0.00050	0.00014	0.40626	0.31031	89.67%	165.39%
NIE	0.00628	0.00122	0.04786	0.03387	54.92%	41.60%
NO	0.00123	0.00052	0.15730	0.25603	68.62%	42.13%
PL	0.00105	0.00062	0.76740	0.76392	179.05%	68.83%
PT	0.00392	0.00050	0.37759	0.45647	12.88%	41.00%
SCO	0.00160	0.00045	0.09316	0.08341	47.98%	93.28%
SK	0.01700	0.00073	0.19061	0.12518	10.97%	74.34%
SI	0.05426	0.02402	0.05237	0.10933	15.70%	5.03%
ES	0.00018	0.00011	0.58485	0.52677	74.11%	66.00%
SE	0.00045	0.00010	0.44715	0.45255	76.61%	115.51%
CH	0.00175	0.00028	0.24021	0.22310	42.66%	66.12%

a specific geographic area of a large country, as some mortality investigations may have been performed just for the larger group. The problem of missing records also emerges when dealing with the oldest ages, as records for this range of ages are often grouped on a multi-year (say, 5-year) basis (in order to avoid to show too erratic death counts). Overall, mortality projections based on the data referring to a small population may then be unreliable and unstable.

As to basis risk, it is a peculiarity of some longevity risk transfer deals. Typically, it occurs because longevity securities are contingent on the mortality of a large population, while they are used to hedge the longevity risk in a small population (namely, a life annuity portfolio or a pension fund).

While basis risk is an issue mainly related to the risk management of post-retirement liabilities, projection accuracy is a broader topic, which (as mentioned above) applies not only to insured populations, but to more general cases. However, we stress that accurate mortality projections are of outstanding importance in view of the risk management of post-retirement benefits. In particular, in order to perform internal assessments or to fulfill solvency requirements for life annuity portfolios or pension funds, it could be necessary to construct a projected life table relying on the mortality experienced within the pool; the risk of inaccuracy is evident because of the (usually) small dimension of the sample referred to.

There is an extensive literature on mortality projections. Most of the contributions concern situations in which data are regular

Table 3
Average values of the ratio $\frac{\text{MSE}^{[R0]}}{\text{MSE}^{[0]}}$.

	Method (f)			Method (m)			Method (Δm)		
	Males	Females	All	Males	Females	All	Males	Females	All
Average value of $\frac{\text{MSE}^{[R0]}}{\text{MSE}^{[0]}}$	124.24%	108.83%	116.53%	67.90%	78.13%	73.02%	60.89%	68.50%	64.69%
# of cases	28	28	56	28	28	56	28	28	56
Average value of $\frac{\text{MSE}^{[R0]}}{\text{MSE}^{[0]}}$, for the cases with $\frac{\text{MSE}^{[R0]}}{\text{MSE}^{[0]}} < 1$	63.41%	66.46%	64.90%	58.91%	60.62%	59.67%	52.28%	56.12%	54.12%
# of cases with $\frac{\text{MSE}^{[R0]}}{\text{MSE}^{[0]}} < 1$	21	20	41	25	20	45	25	23	48
Average value of $\frac{\text{MSE}^{[R0]}}{\text{MSE}^{[0]}}$, for the cases with $\frac{\text{MSE}^{[R0]}}{\text{MSE}^{[0]}} > 1$	306.73%	214.75%	257.67%	142.85%	121.93%	127.64%	132.65%	125.43%	128.14%
# of cases with $\frac{\text{MSE}^{[R0]}}{\text{MSE}^{[0]}} > 1$	7	8	15	3	8	11	3	5	8