Insurance: Mathematics and Economics 54 (2014) 28–40

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

The ruin time under the Sparre-Andersen dual model

Chen Yang *, Kristina P. Sendova

Department of Statistical and Actuarial Sciences, University of Western Ontario, London, Ontario, Canada

HIGHLIGHTS

- We develop general technique of the roots to the Lundberg's equation for dual models.
- An explicit form of the Laplace transform of the time of ruin is obtained.
- The expected dividends of the dual model under the threshold strategy are obtained.

ARTICLE INFO

Article history:
Received March 2013
Received in revised form
October 2013
Accepted 23 October 2013

Keywords:
Generalized Erlang-n innovation times
Generalized Lundberg's equation
Laplace transform
Multiple roots
Sparre-Andersen dual risk model

ABSTRACT

In this paper, we study the Sparre-Andersen dual risk model in which the times between positive gains are independently and identically distributed and have a generalized Erlang-n distribution. An important difference between this model and some other models such as the Erlang-n dual risk model is that the roots to the generalized Lundberg's equation are not necessarily distinct. Hence, we derive an explicit expression for the Laplace transform of the ruin time, which involves multiple roots. Also, we apply our approach for obtaining the expected discounted dividends when the threshold-dividend strategy discussed by Ng (2009) is implemented under the Sparre-Andersen model with Erlang-n distribution of the inter-event times. In particular, we derive an explicit form of the expected discounted dividends when jump sizes are exponential.

© 2013 Elsevier B.V. All rights reserved.

1. Introduction

The dual ruin model is defined as

$$R(t) = u - ct + S(t), \quad t \geq 0, \tag{1.1}$$

where $u > 0$ represents the initial capital, $c > 0$ is the constant expense rate and $\{S(t) : t \geq 0\}$ is the aggregate revenue from time 0 up to time t. This kind of models is widely used in modeling the surplus processes of companies with continuous expense but occasional income due to contingent events (see Avanzi et al., 2007; Ng, 2009; Landriault and Sendova, 2011). One particular case of (1.1) is the compound Poisson dual risk model, which is studied thoroughly in many other papers, including the dividend payment problem with barrier (see Avanzi et al., 2007) or threshold strategy (see Ng, 2009) and the tax payment problem (see Albrecher et al., 2008). Besides, Landriault and Sendova (2011) generalize the Sparre Andersen dual risk model with Erlang-n inter-innovation times by adding a budget-restriction strategy. Recently, in Rodríguez et al. (2013), an explicit form of the Laplace transform of the ruin time under the Erlang-n dual risk model is provided. In this paper, we are mainly interested in the explicit form of the Laplace transform of the time to ruin under the Sparre-Andersen dual model with generalized Erlang-n inter-innovation times. As shown in Ji and Zhang (2012), under the Erlang-n dual risk model, the roots to the Lundberg's equation are distinct. However, under the generalized Erlang-n dual risk model, this is not the case any longer (see Example 5.2). Instead, the multiplicity of the roots should be considered when we derive an explicit form of the Laplace transform of the ruin time.

The contents of this article are organized as follows: Section 2 introduces the notation and model settings. In Section 3, we derive a homogeneous integro-differential equation for an auxiliary quantity related to the Laplace transform of the ruin time. In Section 4, we discuss the number of roots of Lundberg's equation with positive real part in order to find the general solution of the integro-differential equation deduced in Section 3. Section 5 provides the explicit expression of the Laplace transform of the time to ruin. In Section 6, we apply similar arguments for analyzing the threshold-dividend-strategy problem and obtain the explicit form of the expected discounted dividends under the dual risk model with exponential jumps.

2. Notation and model settings

Let the independently and identically distributed (i.i.d.) positive random variables $\{Y_1, Y_2, \ldots\}$ represent the amounts of the

* Corresponding author. Tel.: +1 5196941795.
E-mail addresses: cyang244@uwo.ca (C. Yang), ksendova@stats.uwo.ca (K.P. Sendova).

0167-6687/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.insmatheco.2013.10.012

occasional revenue and denote their common cumulative distribution function (c.d.f.) by $P(y), y \geq 0$, with $P(0) = 0$, their probability density function (p.d.f.) by $p(y) = P'(y), y \geq 0$, and their Laplace transform by $\tilde{p}(s) = \int_0^\infty e^{-sy}\mathrm{d}P(y), s \geq 0$. In (1.1), the renewal process $\{S(t) : t \geq 0\}$ with i.i.d. inter-event times $\{V_i\}_{i=1}^\infty$ is constructed as

$$S(t) = \sum_{i=1}^{N(t)} Y_i,$$

where $N(t) = \max\{k \in \mathbb{N} : V_1 + V_2 + \cdots + V_k \leq t\}$ is the number of gains from time 0 up to time t. By convention, $S(t) = 0$ whenever $N(t) = 0$. In this paper, we assume that the inter-event times $V_j, j = 1, 2, \ldots,$ (we may also call them inter-innovation times) have a generalized Erlang-n distribution with parameters $\lambda_1, \lambda_2, \ldots, \lambda_n > 0$, i.e. V_1, in particular, may be expressed as

$$V_1 \stackrel{d}{=} \sum_{j=1}^n W_j,$$

where W_j is an exponential random variable with mean $1/\lambda_j, j = 1, 2, \ldots, n$. Hence, if we denote the probability distribution function of V_1 by $f(t), t \geq 0$, then the corresponding Laplace transform of $f(t)$ has the form

$$\tilde{f}(s) = \int_0^\infty e^{-st}f(t)\mathrm{d}t = \prod_{j=1}^n \frac{\lambda_j}{\lambda_j + s}, \qquad \mathrm{Re}(s) \geq 0. \tag{2.1}$$

Furthermore, if we define by $f_c(t)$ the p.d.f. of the random variable cV_1, then $f_c(t) = \frac{1}{c}f(t/c)$ and hence by the change of scale property of the Laplace transform, we have

$$\tilde{f}_c(s) = \tilde{f}(cs) = \prod_{j=1}^n \frac{\lambda_j}{\lambda_j + cs}, \qquad \mathrm{Re}(s) \geq 0. \tag{2.2}$$

Now define auxiliary function

$$g_c(t) = e^{-\delta t/c}f_c(t) \tag{2.3}$$

then by the first translation property of the Laplace transform

$$\tilde{g}_c(s) = \tilde{f}_c\left(s + \frac{\delta}{c}\right) = \prod_{j=1}^n \frac{\lambda_j}{\lambda_j + cs + \delta}, \qquad \mathrm{Re}(s) \geq 0. \tag{2.4}$$

Since the dual model describes the surplus process of some kind of business which we do not want to bankrupt with probability 1, we require the so-called *net-profit condition*, namely,

$$c\mathbb{E}[V_1] < \mathbb{E}[Y_1]$$

as one of the basic assumptions for our model. The *net-profit condition* is one of the basic assumptions in many articles related to the dual model such as Avanzi et al. (2007) and Landriault and Sendova (2011). Furthermore, in Section 4, the *net-profit condition* plays an important role in determining the number of roots with positive real part to the generalized Lundberg's equation when there is a simple root on the boundary.

Since the expectation of V_1 is

$$\mathbb{E}[V_1] = \sum_{j=1}^n \mathbb{E}[W_j] = \sum_{j=1}^n \frac{1}{\lambda_j},$$

if we denote by $\mu = \mathbb{E}[Y_1]$, the *net-profit condition* becomes

$$\sum_{j=1}^n \frac{1}{\lambda_j} < \frac{\mu}{c}. \tag{2.5}$$

Now define the ruin time $T := \inf\{t \geq 0 : R(t) = 0\}$ and the ruin probability with given initial capital u

$$\varphi_0(u) = \mathbb{E}[\mathbb{I}(T < \infty)|R(0) = u], \quad u > 0,$$

where $\mathbb{I}(E)$ is the indicator function of an event E. Then

$$\varphi_0(u) < 1$$

for all $u > 0$ only if the *net-profit condition* (2.5) holds. More generally, the Laplace transform of the ruin time, given initial capital u, is defined as

$$\varphi_\delta(u) = \mathbb{E}\left[e^{-\delta T}\mathbb{I}(T < \infty)|R(0) = u\right], \quad u > 0.$$

Our goal is to find an explicit form of $\varphi_\delta(u)$ by solving an integro-differential equation.

In addition, we introduce the Fourier transform of $\varphi_\delta(u)$

$$\hat{\varphi}_\delta(\xi) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty \varphi_\delta(u)e^{-\imath\xi u}\,\mathrm{d}u, \quad \xi \in \mathbb{R},$$

where $\imath = \sqrt{-1}$.

3. An integro-differential equation

In most literature related to ruin theory, the Laplace transform of the ruin time satisfies some integro-differential equation derived by conditioning on the amount and the time of the first innovation. We apply this approach here too. Namely,

$$\varphi_\delta(u) = \int_0^{u/c} e^{-\delta t}\left[\int_0^\infty \varphi_\delta(u - ct + y)\,\mathrm{d}P(y)\right]f(t)\,\mathrm{d}t + \int_{u/c}^\infty e^{-\delta\frac{u}{c}}f(t)\,\mathrm{d}t.$$

With $v = u - ct$ the above equation becomes

$$\begin{aligned}\varphi_\delta(u) &= \frac{1}{c}\int_0^u e^{-\delta\frac{u-v}{c}}\left[\int_0^\infty \varphi_\delta(v + y)\,\mathrm{d}P(y)\right]f\left(\frac{u-v}{c}\right)\mathrm{d}v + e^{-\delta\frac{u}{c}}\overline{F}\left(\frac{u}{c}\right)\\ &= \int_0^u\left[\int_0^\infty \varphi_\delta(v + y)\,\mathrm{d}P(y)\right]g_c(u - v)\,\mathrm{d}v + e^{-\delta\frac{u}{c}}\overline{F}\left(\frac{u}{c}\right),\end{aligned}$$

where $\overline{F}(t)$ is the tail distribution of the density function $f(t)$. Hence, $\varphi_\delta(u)$ satisfies a convolution-type integro-differential equation of the form

$$\zeta(u) = \int_0^u \mathcal{I}[\zeta](v)g_c(u - v)\,\mathrm{d}v + G(u) \tag{3.1}$$

with $G(t) = e^{-\delta t/c}\overline{F}(t/c)$ and operator $\mathcal{I} : C_{(0,\infty)} \mapsto C_{(0,\infty)}$ defined as

$$\mathcal{I}[\zeta](u) = \int_0^\infty \zeta(u + y)\,\mathrm{d}P(y).$$

For integro-differential equation (3.1), we have the following theorem for a particular class of functions $G(u)$.

Theorem 3.1. *For a function*

$$G(u) \in \left\{h \in C^n_{(0,\infty)} : \left[\prod_{j=1}^n\left(\lambda_j + \delta + c\frac{\mathrm{d}}{\mathrm{d}u}\right)\right]h(u) = 0\right\}, \tag{3.2}$$

if $\zeta(u)$ satisfies (3.1), then $\zeta(u)$ also satisfies the homogeneous integro-differential equation

$$\left[\prod_{j=1}^n\left(\lambda_j + \delta + c\frac{\mathrm{d}}{\mathrm{d}u}\right)\right]\zeta(u) = \left(\prod_{j=1}^n \lambda_j\right)\mathcal{I}[\zeta](u), \quad u > 0, \tag{3.3}$$

with boundary conditions $\zeta^{(i)}(0) = G^{(i)}(0)$ for $i = 0, 1, \ldots, n - 1$.

Download English Version:

https://daneshyari.com/en/article/5076776

Download Persian Version:

https://daneshyari.com/article/5076776

Daneshyari.com