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A Markov-modulated jump-diffusion risk model with randomized observation periods and threshold dividend strategy ☆

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ABSTRACT

This paper considers a Markov-modulated jump-diffusion risk model with randomized observation periods and threshold dividend. A second order integro-differential system of equations that characterizes the expected discounted dividend payments is obtained. As a closed-form solution does not exist, a numerical procedure based on the sinc function approximation through a collocation method is proposed. Finally, an example illustrating the procedure is presented.

1. Introduction

Dividend strategies for an insurance risk model were first proposed by De Finetti (1957) to model in a more realistic way an insurer's surplus. From then on, the barrier and threshold dividend strategies have been studied for a variety of insurance risk model, for example Asmussen and Taskar (1997), Cai et al. (2006), Dickson and Waters (2004), Lin et al. (2003). The above continuous-time models of the surplus process need continuous observation of the surplus process, which cannot be realized in practice. Albrecher et al. (2011a) proposed a compound Poisson model with stochastic observation periods and constant dividend barrier, and presented some result on the distribution of dividend payment until ruin. The idea of randomized observation periods means the risk process can be "looked" only at random times (called observation times), which has been proposed and discussed by Albrecher et al. (2013, 2011a,b). The risk model with stochastic observations is still a continuous-time model for the surplus, but to assume that observations are only possible at discrete points in time. It is a computational bridge between the continuous-time and the discrete-time collective risk models that still enables explicit expressions.

The Markov-modulated risk model was proposed by Asmussen (1989), in which the ruin probability was studied. The model is also called Markovian regime switching model in the finance and the actuarial science literature. This model can capture the feature that insurance policies may need to change if economical or political environment changes. Recently, there have been resurgent interests of using regime switching models in the finance and actuarial science; see Avanzi et al. (2012), Diko and Usábel (2011), Ng and Yang (2006). In this paper, motivated by the study of Albrecher et al. (2011a) and Asmussen (1989) we investigate some corresponding results in a Markov-modulated jump-diffusion risk model with randomized observation periods and threshold dividend strategies.

Let $\{J_t\}_{t\geq 0}$ be a homogeneous continuous-time Markov chain taking values in a finite set $\mathbb{M} = \{1, 2, \ldots, d\}$ with generator $\Lambda = (\lambda_{ij})$. Λ is assumed to be irreducible with stationary distribution $\pi = (\pi_1, \pi_2, \ldots \pi_d)$. We identify the state space of the chain as a finite set of unit vectors $E := \{e_1, e_2, \ldots, e_d\}$ without loss of generality, where $e_i \in \mathbb{R}^d$ and the jth component of e_i is the Kronecker delta δ_{ij}, for each $i, j = 1, 2, \ldots, d$. The set E is called the canonical state space of the chain.

The risk process $\{R(t)\}_{t\geq 0}$ is given by

$$R(t) = x + \sum_{i=1}^{d} \int_0^t I_{\{J_s = e_i\}} \mathrm{d}R_i(s), \quad t \geq 0, \tag{1}$$

where $x = R(0) \geq 0$ is the initial surplus level, $I_{\{.\}}$ denotes the indicator function and $\{R_1(t)\}, \{R_2(t)\}, \ldots, \{R_d(t)\}$ are d independent

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risk processes defined as follows,

$$R_i(t) = x\int_0^t e^{r_i(t-s)}\mathrm{d}s + c_i\int_0^t e^{(t-s)r_i}\mathrm{d}s + \sigma_i\int_0^t e^{(t-s)r_i}\mathrm{d}W(s) - \sum_{k=1}^{N_i(t)} e^{(t-\sum_{j=1}^{k} S_j^i)r_i}Y_k^i, \quad (2)$$

where c_i is the constant premium income rate and σ_i is the constant volatility. $\{W(t)\}$ is a standard Brownian motion. $\{N_i(t)\}$ is a homogeneous Poisson process with rate β_i defined as $N_i(t) = sup\{k : S_1^i + S_2^i + \cdots + S_k^i \le t\}$, where $S_k^i, k = 1, 2, \ldots$ are the i.i.d inter-times of $R_i(t)$ with exponential distribution. The claim sizes $\{Y_n^i, n = 1, 2, \ldots\}$ are independent and identically distributed as a generic continuous random variable (r.v.) with c.d.f. $F_Y^i(.)$, p.d.f. $f_Y^i(.)$ and mean μ_i. $r_i > 0$ is the constant force of interest. From the above, the condition of having a positive expected profit is

$$\sum_{i=1}^{d}\pi_i(c_i - \beta_i\mu_i) > 0.$$

Let $\{Z_k\}_{k=1}^{+\infty}$ denote the observation times and Z_{-1} be the last observation time before $Z_0 = 0$. If time 0 is an observation point, $Z_{-1} = Z_0 = 0$. Let $T_k = Z_k - Z_{k-1} (k = 1, 2, \ldots)$ be the kth time interval between observations, and assume that $\{T_k\}_{k=1}^{+\infty}$ is an i.i.d sequence distributed as a r.v. T, which has a common exponential distribution with parameter γ. In addition, $\{Y_k^i, k = 1, 2, \ldots, i = 1, \ldots, d\}$, $\{N_i(t), i = 1, 2, \ldots, d\}$, $\{W(t), t \ge 0\}$, $\{Z_k, i = 1, 2, \ldots\}$ and $\{J_t, t \ge 0\}$ are all independent. At the observation times $\{Z_k, k = 1, 2, \ldots\}$, if the current surplus level $R(Z_k) = x$ exceeds the barrier level b, dividends will be paid continuously at a constant rate $\alpha (0 < \alpha \le c)$ during the $k + 1$th time interval between observations. If $0 \le R(Z_k) = x \le b$, no dividends are paid during the $k + 1$th time interval between observations. And the process will be declared ruined if $R(Z_k) = x \le 0$. In particular that ruin can now only be observed at these random observation times and so a surplus level below 0 between observation points will only result in actual ruin if it is also negative at the next observation time. With the above-defined dividend rule with barrier b, denote the sequence of surplus levels at the time points $\{Z_k\}_{k=1}^{+\infty}$ by $\{U_b(k)\}_{k=1}^{+\infty}$. If time 0 is not an observation time, we assume paying dividends continuously at rate α or not depends on the value of $U_b(-1)$ which is the value of surplus at the last observation time before time 0 and provides the latest information of the surplus process. For the case of $U_b(-1) > b$, dividends are paid continuously at rate α between time Z_0 and time Z_1, while if $0 \le U_b(-1) \le b$, no dividend is paid between time Z_0 and time Z_1.

Let $c = (c_1, c_2, \ldots, c_d), \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_d)$, and $r = (r_1, r_2, \ldots, r_d)$. Define $c(t) = \langle c, J_t\rangle, \sigma(t) = \langle \sigma, J_t\rangle$ and $r(t) = \langle r, J_t\rangle$. Where $\langle , \rangle$ is the scalar product in $\mathbb{R}^d$. Denote $S^i(t) = \sum_{k=1}^{N_i(t)} Y_k^i$. With the initial surplus level $U_b(0) = x, x \in \mathbb{R}$, we then have the relationship (Eqs. (3) and (4) is given in Box I).

The time of ruin is defined by $\tau_b = Z_{k_b}$, where $k_b = inf\{k \ge 0 : U_b(k) < 0\}$ is the number of observation intervals before ruin. Let $\Delta_\delta(x; b)$ be the cumulative amount of dividend paid out up to time τ_b for a discounted rate $\delta \ge 0$, then

$$\Delta_\delta(x; b) = \begin{cases} \displaystyle\sum_{k=1}^{k_b-1} \alpha e^{-\delta Z_k} I_{\{U_b(k)>b\}} \cdot \bar{a}_{\overline{T_{k+1}}|\delta} + \alpha \bar{a}_{\overline{T_1}|\delta}, \\ \qquad x = U_b(0) \in \mathbb{R}, U_b(-1) > b, \\ \displaystyle\sum_{k=1}^{k_b-1} \alpha e^{-\delta Z_k} I_{\{U_b(k)>b\}} \cdot \bar{a}_{\overline{T_{k+1}}|\delta}, \\ \qquad x = U_b(0) \in \mathbb{R}, U_b(-1) \le b, \end{cases} \quad (5)$$

where $\bar{a}_{\bar{t}|\delta}$ is the present value of a continuous annuity, $\bar{a}_{\bar{t}|\delta} = \frac{1-e^{-\delta t}}{\delta}$ and $I_{\{.\}}$ is the indicator function.

We study the expected discounted dividend payments until ruin for the discounted rate $\delta \ge 0$. Let $P_i(.) = P(.|J_0 = i)$. Since the conditioning technique exploits the removal of the time stamp, we will need to consider the definition (5) Δ_δ for all $x \in \mathbb{R}$, where now time 0 is a priori not an observation time. It is easy to see that $\Delta_\delta(x; b)$ behaves differently with different values of $U_b(-1)$. Hence, we set two functions $V_{1,i}(x; b)$ and $V_{2,i}(x; b)$ to denote the conditional expectations of $\Delta_\delta(x; b)$ under P_i with different values of $U_b(-1)$, that are

$$\begin{cases} V_{1,i}(x; b) = E_i[\Delta_\delta(x; b)|U_b(0) = x, U_b(-1) > b], \\ V_{2,i}(x; b) = E_i[\Delta_\delta(x; b)|U_b(0) = x, 0 \le U_b(-1) \le b], \end{cases} \quad (6)$$

which are the main quantities of interest in this paper.

In the Markov-modulated model, we obtain a second order integro-differential system of equations that characterizes the function of the expected discounted dividend payments. The closed-form solution of the integro-differential equation system does not exist; a numerical procedure based on the sinc function through a collocation method is proposed. The sinc method is a highly efficient numerical method developed by Frank Stenger, the pioneer of this field, people in his school and others (Stenger, 1993; Lund and Bowers, 1991; Stenger, 1976, 2000, 2011). It has been widely used in various fields of numerical analysis such as interpolation, quadrature, approximation of transforms, solution of integral, and ordinary differential and partial differential equations. In Chen and Ou (2013) the sinc method was proposed for a Fredholm–Volterra integro-differential equation to calculate the value of the expected discounted dividend function in a compound Poisson risk model with proportional investment. The authors approximated the expected discounted dividend function over $(0, \infty)$ having algebraic decay for large and small values of the argument. In this work, the sinc-collocation method is developed for solving the linear system of integro-differential equations of Fredholm–Volterra type. We approximate the expected discounted dividend functions over $(-\infty, +\infty)$ having algebraic decay at $\pm\infty$. As an example illustrating the sinc procedure, we study the effect of randomized observation times and environment states on the total discounted dividend payments until ruin.

We organize our paper as follows. In Section 2, the integro-differential system that characterizes the expected discounted dividend payments is derived. In Section 3, a numerical method to approximate the solution of the system via the sinc-collocation method is considered. In Section 4 we give a numerical example to illustrate the impacts of the economical or political environment on the expected discounted dividend payments. The final section concludes the paper.

2. Integro-differential system for the expected discounted dividend payments

In this section, we derive the integro-differential system of equations satisfied by $\{V_{k,i}(x; b), i = 1, 2, \ldots, d, k = 1, 2\}$.

Theorem 1. $\{V_{k,i}(x; b), i = 1, 2, \ldots, d, k = 1, 2\}$ *satisfy the following system of integro-differential equations:*

$$\frac{1}{2}\sigma_i^2 V_{1,i}''(x; b) + (r_i x + c_i - \alpha)V_{1,i}'(x; b) - (\delta + \beta_i + \gamma I_{\{x\le b\}} - \lambda_{ii})V_{1,i}(x; b) + \sum_{j\ne i}\lambda_{ij}V_{1,j}(x; b) + \gamma I_{\{0\le x\le b\}}V_{2,i}(x; b) + \beta_i\int_0^\infty V_{1,i}(x-y; b)f_Y^i(y)\mathrm{d}y + \alpha = 0, \quad (7)$$