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Asset allocation for a DC pension fund with stochastic income and mortality risk: A multi-period mean–variance framework☆

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HIGHLIGHTS

- A multi-period mean–variance asset allocation for DC pension funds is studied.
- Stochastic income and mortality risk are considered in the model.
- Lagrange multiplier method and dynamic programming approach are used.
- Explicit expressions of the efficient strategy and efficient frontier are derived.
- Some special cases are discussed and some numerical analyses are presented.

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ABSTRACT

This paper investigates an asset allocation problem for defined contribution pension funds with stochastic income and mortality risk under a multi-period mean–variance framework. Different from most studies in the literature where the expected utility is maximized or the risk measured by the quadratic mean deviation is minimized, we consider synthetically both to enhance the return and to control the risk by the mean–variance criterion. First, we obtain the analytical expressions for the efficient investment strategy and the efficient frontier by adopting the Lagrange dual theory, the state variable transformation technique and the stochastic optimal control method. Then, we discuss some special cases under our model. Finally, a numerical example is presented to illustrate the results obtained in this paper.

1. Introduction

According to the fund procurement and operation pattern, pension funds can mainly be divided into two types: one is defined benefit (DB) pension funds, the other is defined contribution (DC) pension funds. A DB pension fund is a pension fund where the benefits are set in advance by the sponsor. In such a fund, contributions are set and constantly adjusted so as to ensure that the fund remains in balance, and the risk is borne by the organizer of the fund. A DC pension fund is a pension fund where contributions are set and benefits therefore depend solely on the accumulation scale and the return of the investment, thus the financial risk is borne by the members.

Historically, DB pension funds have been the more popular and preferred by workers since the management is easy and the risk is borne by the sponsors of the fund. However, due to the demographic evolution and the development of the capital market, especially due to the population aging problem and the longevity risk, DC has become more and more popular in the global pension market in recent years, and more and more countries have shifted completely or partially from a DB pension scheme to a DC pension scheme. As a result, the study of DC pension fund investment management has become a research hot topic in the actuarial and the financial literature over the past decade. Due to space limitations, only some important research results are briefly introduced below.

By minimizing the risk measured by the quadratic mean deviation (also called the target-based criterion), Vigna and

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Haberman (2001) and Haberman and Vigna (2002) investigate multi-period DC pension fund investment managements during accumulation phase, and obtain the optimal investment strategies by using the dynamic programming method. Gerrard et al. (2004) extend the work of Vigna and Haberman (2001) to continuous-time setting and post-retirement case. Based on continuous-time expected utility maximization: Deelstra et al. (2003) and Giacinto et al. (2011) investigate optimal asset allocation for DC pension funds with stochastic interest rates and a minimum guarantee protection; Gao (2009) investigate the optimal portfolios for DC pension funds under a constant elasticity of variance (CEV) by applying the Legendre transform and dual theory; Han and Hung (2012) consider a portfolio selection problem for DC pension fund under inflation by using the stochastic control approach. Using quadratic risk minimization criterion, Gerrard et al. (2012) formulate and solve a stochastic control and optimal stopping problem of finding the optimal time of annuitization for a DC pension scheme in the de-cumulation phase. Based on target-driven framework and prospect theory in behavioral finance, Blake et al. (2013) study the optimal asset allocation problem for DC pension funds.

As the contribution period in a pension fund is very long, generally from 20 to 40 years due to promotion prospect and changes in economic environment, it is crucial to allow a stochastic term structure for the income (salary). As the contribution is often a fixed percentage of salary, the contribution is also stochastic. Therefore, it is crucial to take into account the income risk for a DC pension fund management. Recently, by using continuous-time utility maximization model, Cairns et al. (2006), Zhang and Ewald (2010) and Ma (2011) investigate the optimal asset allocation with stochastic income for DC pension funds in different market settings, where the income process is described by a geometric Brownian motion. Under the DC pension framework, Emms (2012) study the lifetime investment and consumption problem with stochastic income based on CARA utility maximization model.

A pension fund member may die before his retirement. In such a case, his pension plan has to be terminated due to mortality risks, such as traffic accident, fire hazard and serious illnesses, etc. Although there are many studies in the literatures on ordinary portfolio selection and life insurance problems with uncertain time horizon, see for example, Martellini and Urosevic (2006), Pliska and Ye (2007) and Christophette et al. (2008), there are very limited studies aim at pension fund investment managements. By maximizing the expected utility, Charupat and Milevsky (2002) consider the optimal asset allocation in a variable annuity contract that may have some relation to DC pension fund; Hainaut and Devolder (2007) consider the optimal dividend policy and the asset allocation of a DB pension fund under the mortality risk. By minimizing the risk which is a quadratic target-based cost function, Hainaut and Deelstra (2011) investigate optimal contribution rate of a DB pension fund with a stochastic mortality which is modeled by a jump process. But all these studies focus on annuity contract and DB pension fund management problems. Research on DC pension fund portfolio selection problems with mortality risks is very limited.

Furthermore, researchers of the above mentioned studies either maximize the expected utility or minimize the risk which is measured by the quadratic mean deviation, but they do not apply the mean-risk bi-objective criteria to study the pension fund investment problems. It is well known that the mean-risk framework has become one of the most basic frameworks in the modern portfolio selection theory since Markowitz (1952) published his seminal work on the mean–variance portfolio selection. The mean–variance criterion is a mean-risk bi-objective criterion that strikes a balance between enhancing the return and controlling the risk, where the risk is measured by the variance of the portfolio's return. Along with the break through of solving of the dynamic mean–variance model (see Zhou and Li (2000) and Li and Ng (2000) for continuous-time case and multi-period case, respectively), the dynamic mean–variance model also become one of the most important frameworks to study asset allocation and asset–liability management problems, see, for example, Fu et al. (2010), Costa and Oliveira (2012), Chen et al. (2008), Wu and Li (2011), Chiu and Wong (2013) and Yao et al. (2013a,b). But, to our knowledge, only very few studies in the literature apply the dynamic mean–variance models to study pension fund investment problems. By using the continuous-time mean–variance model, Delong et al. (2008) and Josa-Fombellida and Rincón-Zapatero (2008) study the optimal investment and contribution strategies for DB pension funds; Vigna (2012) investigates the portfolio selection problem for DC pension funds. But they only focus on the continuous-time case without touching the multi-period case.

To the best of our knowledge, no research work in the literature investigates the multi-period version of DC pension fund asset allocation problems by using the mean–variance framework. This paper attempts to explore this topic. In this paper, we consider a DC pension fund investment management problem by using the multi-period mean–variance model. In addition, we incorporate stochastic income and mortality risk into our model. Mathematically, the inclusion of stochastic income and mortality risk increases the difficulty in solving the Bellman equation which comes from the dynamic programming approach. Specifically, the inclusion of the stochastic income adds a new state variable to the model, namely, there are two state variables for our problem: the wealth and the wage income. Furthermore, after incorporating the mortality risk, our asset allocation problem for a DC pension fund becomes more complicated. Therefore, adding the stochastic income and the mortality risk into the multi-period mean–variance model drastically increase the computational complexity in obtaining the closed form solutions. These solutions cannot be obtained easily by using the embedding technique used in Li and Ng (2000) since the calculation procedure of the embedding technique is quite troublesome. Different from most of the researches in the literature on multi-period mean–variance models which adopt the embedding technique, we use the state variable transformation technique and the Lagrange dual theory to solve the model synthetically. Compared to the embedding technique, our approach is relatively simple in procedure settings and calculations.

The remainder of the paper is organized as follows. The market setting is described in Section 2. The multi-period mean–variance asset allocation model for DC pension funds with stochastic income and mortality risk are set up in the same section. In Section 3, the original model is transformed into a standard multi-period stochastic control problem by using the Lagrange multiplier, then the corresponding analytical solution is derived by using the state variable transformation technique and the dynamic programming approach. Closed form expressions for the efficient investment strategy and the efficient frontier are obtained in Section 4 by using the Lagrange dual theory. Some special cases are discussed in Section 5. In Section 6, some numerical analyses are presented to illustrate our results. The paper is concluded in Section 7.

2. Model formulation

Consider a financial market consisting of $n+1$ assets that may include a risk free asset or be all risky. Denote by $e_k = (e_k^0, e_k^1, e_k^2, \ldots, e_k^n)'$ the random returns of these $n+1$ assets over period k (i.e., the time interval from time k to time $k+1$), $k = 0, 1, \ldots, T-1$, where $'$ represents the transpose of a matrix or a vector. This paper considers a multi-period asset allocation problem for a DC pension fund. Suppose that a representative