Insurance: Mathematics and Economics 54 (2014) 133–143

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Optimal dividends in the dual model under transaction costs

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ARTICLE INFO

Article history:
Received March 2013
Received in revised form
November 2013
Accepted 11 November 2013

JEL classification:
C44
C61
G24
G32
G35

MSC:
60G51
93E20

Keywords:
Dual model
Dividends
Impulse control
Spectrally positive Lévy processes
Scale functions

ABSTRACT

We analyze the optimal dividend payment problem in the dual model under constant transaction costs. We show, for a general spectrally positive Lévy process, an optimal strategy is given by a (c_1, c_2)-policy that brings the surplus process down to c_1 whenever it reaches or exceeds c_2 for some $0 \leq c_1 < c_2$. The value function is succinctly expressed in terms of the scale function. A series of numerical examples are provided to confirm the analytical results and to demonstrate the convergence to the no-transaction cost case, which was recently solved by Bayraktar et al. (2013).

1. Introduction

We solve the optimal dividend problem under fixed transaction costs in the so-called *dual model*, in which the surplus of a company is driven by a Lévy process with positive jumps (*spectrally positive Lévy* process). This is an appropriate model for a company driven by inventions or discoveries. The case without transaction costs has recently been well-studied; see Avanzi et al. (2007), Bayraktar and Egami (2008), Avanzi and Gerber (2008), and Avanzi et al. (2011). In particular, in Bayraktar et al. (2013), we show the optimality of a barrier strategy (reflected Lévy process) for a general spectrally positive Lévy process of bounded or unbounded variation.

A strategy is assumed to be in the form of impulse control; whenever dividends are accrued, a constant transaction cost $\beta > 0$ is incurred. As opposed to the barrier strategy that is typically optimal for the no-transaction cost case, we shall pursue the optimality of the so-called (c_1, c_2)-policy that brings the surplus process down to c_1 whenever it reaches or exceeds c_2 for some $0 \leq c_1 < c_2 < \infty$. While, as in Loeffen (2009), Thonhauser and Albrecher (2011), an optimal strategy may not lie in the set of (c_1, c_2)-policies for the spectrally negative Lévy case, we shall show that it is indeed so in the dual model for any choice of underlying spectrally positive Lévy process. As a related work, we refer the reader to a compound Poisson dual model by Yao et al. (2011) where transaction costs are incurred for capital injections. In inventory control, the optimality of similar policies, called (s, S)-policies, is shown to be optimal in Benkherouf and Bensoussan (2009), Bensoussan et al. (2005) for a mixture of a Brownian motion and a compound Poisson process and in Yamazaki (2013) for a general spectrally negative Lévy process.

Following Bayraktar et al. (2013), we take advantage of the fluctuation theory for the spectrally positive Lévy process (see e.g. Bertoin (1996), Doney (2007) and Kyprianou (2006)). The expected net present value (NPV) of dividends (minus transaction costs) under a (c_1, c_2)-policy until ruin is first written in terms of the scale function. We then show the existence of the maximizers

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0167-6687/$ – see front matter
http://dx.doi.org/10.1016/j.insmatheco.2013.11.007

$0 \leq c_1^* < c_2^* < \infty$ that satisfy the continuous fit (resp. smooth fit) at c_2^* when the surplus process is of bounded (resp. unbounded) variation and that the derivative at c_1^* is one when $c_1^* > 0$ and is less than or equal to one when $c_1^* = 0$. These properties are used to verify the optimality of the (c_1^*, c_2^*)-policy.

In order to evaluate the analytical results and to examine the connection with the no-transaction cost case developed by Bayraktar et al. (2013), we conduct a series of numerical experiments using Lévy processes with positive i.i.d. phase-type jumps with or without a Brownian motion (Asmussen et al., 2004). We shall confirm the existence of the maximizers $0 \leq c_1^* < c_2^* < \infty$ and examine the shape of the value function at c_1^* and c_2^*. We further compute for a sequence of unit transaction costs and confirm that, as $\beta \downarrow 0$, the value function as well as c_1^* and c_2^* converges to the ones obtained for the no-transaction cost case in Bayraktar et al. (2013).

The rest of the paper is organized as follows. Section 2 gives a mathematical model of the problem. In Section 3, we compute the expected NPV of dividends under the (c_1, c_2)-policy via the scale function. Section 4 shows the existence of $0 \leq c_1^* < c_2^* < \infty$ that maximize the expected NPV over c_1 and c_2. Section 5 verifies the optimality of the (c_1^*, c_2^*)-policy. We conclude the paper with numerical results in Section 6.

2. Mathematical formulation

We will denote the surplus of a company by a spectrally positive Lévy process $X = \{X_t; t \geq 0\}$ whose *Laplace exponent* is given by

$$\psi(s) := \log \mathbb{E}\left[e^{-sX_1}\right] = cs + \frac{1}{2}\sigma^2 s^2 + \int_{(0,\infty)} (e^{-sz} - 1 + sz 1_{\{0<z<1\}})\nu(\mathrm{d}z), \quad s \geq 0 \tag{2.1}$$

where ν is a Lévy measure with the support $(0, \infty)$ that satisfies the integrability condition $\int_{(0,\infty)} (1 \wedge z^2)\nu(\mathrm{d}z) < \infty$. It has paths of bounded variation if and only if $\sigma = 0$ and $\int_{(0,1)} z\,\nu(\mathrm{d}z) < \infty$. In this case, we write (2.1) as

$$\psi(s) = \mathfrak{d}s + \int_{(0,\infty)} (e^{-sz} - 1)\nu(\mathrm{d}z), \quad s \geq 0$$

with $\mathfrak{d} := c + \int_{(0,1)} z\,\nu(\mathrm{d}z)$; the resulting drift of the process is $-\mathfrak{d}$. We exclude the trivial case in which X is a subordinator (i.e., X has monotone paths a.s.). This assumption implies that $\mathfrak{d} > 0$ when X is of bounded variation.

Let $\mathbb{P}_x$ be the conditional probability under which $X_0 = x$ (also let $\mathbb{P} \equiv \mathbb{P}_0$), and let $\mathbb{F} := \{\mathcal{F}_t : t \geq 0\}$ be the filtration generated by X. Using this, the drift of X is given by

$$\mu := \mathbb{E}[X_1] = -\psi'(0+). \tag{2.2}$$

In order to make sure the problem is non-trivial and well-defined, we assume throughout the paper that this is finite.

Assumption 2.1. We assume that $\mu \in (-\infty, \infty)$.

A (dividend) *strategy* $\pi := \left\{L_t^\pi; t \geq 0\right\}$ is given by a non-decreasing, right-continuous and $\mathbb{F}$-adapted *pure jump* process starting at zero in the form $L_t^\pi = \sum_{0 \leq s \leq t} \Delta L_s^\pi$ with $\Delta L_t = L_t - L_{t-}, t \geq 0$. Corresponding to every strategy π, we associate a *controlled surplus* process $U^\pi = \{U_t^\pi : t \geq 0\}$, which is defined by

$$U_t^\pi := X_t - L_t^\pi, \quad t \geq 0,$$

where $U_{0-}^\pi = x$ is the initial surplus and $L_{0-}^\pi = 0$. The time of ruin is defined to be

$$\sigma^\pi := \inf\left\{t > 0 : U_t^\pi < 0\right\}.$$

A lump-sum payment cannot be more than the available funds and hence it is required that

$$\Delta L_t^\pi \leq U_{t-}^\pi + \Delta X_t, \quad t \leq \sigma^\pi \text{ a.s.} \tag{2.3}$$

Let Π be the set of all admissible strategies satisfying (2.3). The problem is to compute, for $q > 0$, the expected NPV of dividends until ruin

$$v_\pi(x) := \mathbb{E}_x\left[\int_0^{\sigma^\pi} e^{-qt}\mathrm{d}\left(L_t^\pi - \sum_{0\leq s\leq t} \beta 1_{\{\Delta L_s^\pi > 0\}}\right)\right], \quad x \geq 0,$$

where $\beta > 0$ is the unit transaction cost, and to obtain an admissible strategy that maximizes it, if such a strategy exists. Hence the (optimal) value function is written as

$$v(x) := \sup_{\pi \in \Pi} v_\pi(x), \quad x \geq 0. \tag{2.4}$$

3. The (c_1, c_2)-policy

We aim to prove that a (c_1^*, c_2^*)-policy is optimal for some $c_2^* > c_1^* \geq 0$. For $c_2 > c_1 \geq 0$, a (c_1, c_2)-policy, $\pi_{c_1,c_2} := \left\{L_t^{c_1,c_2}; t \geq 0\right\}$, brings the level of the controlled surplus process $U^{c_1,c_2} := X - L^{c_1,c_2}$ down to c_1 whenever it reaches or exceeds c_2. Let us define the corresponding expected NPV of dividends as

$$v_{c_1,c_2}(x) := \mathbb{E}_x\left[\int_0^{\sigma_{c_1,c_2}} e^{-qt}\mathrm{d}\left(L_t^{c_1,c_2} - \sum_{0\leq s\leq t} \beta 1_{\{\Delta L_s^{c_1,c_2} > 0\}}\right)\right], \quad x \geq 0, \tag{3.1}$$

where $\sigma_{c_1,c_2} := \inf\left\{t > 0 : U_t^{c_1,c_2} < 0\right\}$ is the corresponding ruin time. In this section, we shall express these in terms of the scale function.

3.1. Scale functions

Fix $q > 0$. For any spectrally positive Lévy process, there exists a function called the *q-scale function*

$$W^{(q)} : \mathbb{R} \to [0, \infty),$$

which is zero on $(-\infty, 0)$, continuous and strictly increasing on $[0, \infty)$, and is characterized by the Laplace transform:

$$\int_0^\infty e^{-sx} W^{(q)}(x)\mathrm{d}x = \frac{1}{\psi(s) - q}, \quad s > \Phi(q),$$

where

$$\Phi(q) := \sup\{\lambda \geq 0 : \psi(\lambda) = q\}.$$

Here, the Laplace exponent ψ in (2.1) is known to be zero at the origin and convex on $[0, \infty)$; therefore $\Phi(q)$ is well-defined and is strictly positive as $q > 0$. We also define, for $x \in \mathbb{R}$,

$$\overline{W}^{(q)}(x) := \int_0^x W^{(q)}(y)\mathrm{d}y,$$

$$Z^{(q)}(x) := 1 + q\overline{W}^{(q)}(x),$$

$$\overline{Z}^{(q)}(x) := \int_0^x Z^{(q)}(z)\mathrm{d}z = x + q\int_0^x \int_0^z W^{(q)}(w)\mathrm{d}w\mathrm{d}z.$$

Notice that because $W^{(q)}$ is uniformly zero on the negative half-line, we have

$$Z^{(q)}(x) = 1 \quad \text{and} \quad \overline{Z}^{(q)}(x) = x, \quad x \leq 0. \tag{3.2}$$