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Fuzzy risk adjusted performance measures: Application to hedge funds

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ABSTRACT

In this paper, following the notion of probabilistic risk adjusted performance measures, we introduce that of fuzzy risk adjusted performance measures (FRAPM). In order to deal efficiently with the closing-based returns bias induced by market microstructure noise, as well as to handle their uncertain variability, we combine fuzzy set theory and probability theory. The returns are first represented as fuzzy random variables and then used in defining fuzzy versions of some adjusted performance measures. Using a recent ordering method for fuzzy numbers, we propose a ranking of funds based on these fuzzy performance measures. Finally, empirical studies carried out on fifty French hedge funds confirm the effectiveness and give the benefits of our approach over the classical performance ratios.

1. Introduction

A hedge fund can be defined as a "pooled investment vehicle that is privately organized, administered by professional investment managers, and not widely available to the public".[1] Due to their private nature and because they are not subjected to several requirements of regulatory bodies, there is a lack of transparency of the hedge fund managers' activities. As hedge fund managers have the possibility to not disclose their performance, daily published results are often subject to bias (Eling, 2006). The use of such biased data for systematic risk (beta) estimation tends to lead to inconsistent ordinary least squares estimators in linear pricing models such as the Capital Asset Pricing Model (CAPM) or the Arbitrage Pricing Theory (APT).[2] Generally speaking, for linear regression models with measurement errors in the regressors estimated by the ordinary least squares method, Cragg (1994) demonstrated that the slope coefficients were biased toward zero and concluded that the measurement error "produced a bias of the opposite sign on the intercept coefficient when the average value of the explanatory variables is positive". It follows that the presence of noise in the return biases the estimates of the systematic risk beta and of Jensen's alpha, leading to the prominence of the performance evaluation based on the linear factor market models.

Moreover, the assumption of linearity of the causal relationship of returns with a set of covariates usually referred to as risk factors as well as that of the normal distribution of financial asset returns formulated by seminal researchers (Markowitz, Sharpe, Treynor, etc) in quantitative finance, have been extensively discussed in the literature in recent years. For the special case of hedge funds, these two assumptions are widely violated as shown in Agarwal and Naik (2000) and Mitchell and Pulvino (2001) and empirically confirmed by Amin and Kat (2003). The violation of these two assumptions implies invalidity of the CAPM for hedge funds performance evaluation. Hence the use of the traditional adjusted performance measures becomes questionable. This conclusion and other similar ones have motivated some authors such as Capocci and Hübner (2004), Coën and Hübner (2009) and Darolles and Gourieroux (2010) to propose alternatives to the adjusted performance measures derived from Sharpe's market line based on probability theory.

In this paper, we focus on hedge funds performance evaluation. We propose combining fuzzy set theory and probability theory to construct some adjusted performance measures. Our modeling approach aims to deal with imprecision induced by market microstructure noise and the stochastic variability of the risk factors. As explained by Shapiro (2009), these two sources of the uncertainty can both be modeled by a fuzzy random variable. For this purpose, the basic assumption of our modeling approach is the representation of financial asset returns through a fuzzy random variable. Note that the fuzzy representation of financial asset returns has been resorted to in the literature by many authors including, among others, Tanaka and Guo (1999), Smimou et al. (2008) and Yoshida (2009). For an overview of some applications

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[1] A definition adapted from Amin and Kat (2003).

[2] Klepper and Leamer (1984) and Leamer (1987), among others, provide evidence of inconsistency of ordinary least squares estimators in linear regression models with measurement errors in the regressors.

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of fuzzy logic in insurance, see Shapiro (2004) and some references therein.

This paper studies the fuzziness of returns over a period, as the effect of noise induced by the market microstructure frictions on the observed returns. Our fuzzy set-valued returns can be seen as a generalized form of the interval-valued and the real-valued one. This approach aims at determining adjusted performance measures by taking into account the imprecision of the risk factors. We will first focus on the estimation of the market line by considering the returns as fuzzy random variables. This question was the subject of one of our recent reflections which will be referred to as Mbairadjim Moussa et al. (2012) hereafter.

The remainder of this paper is organized as follows. Section 2 is a brief presentation of basic concepts of fuzzy set theory necessary to the introduction of the fuzzy presentation process of monthly returns in Section 3. Section 4 is devoted to the definition and mathematical characterization of fuzzy adjusted risk measures. In Section 5, we review an ordering method for fuzzy number which can be applied in order to produce FRAPM based funds ranking. An application to hedge funds data from France is given in Section 6. We determine and compare the rankings associated with the classical crisp performance measures and the fuzzy adjusted performance measures. Finally some conclusions are listed in Section 7.

2. Preliminaries

Before proceeding to formal presentation of fuzzy adjusted performance measures, we first briefly review three of the basic concepts of fuzzy theory; namely fuzzy sets, fuzzy numbers and fuzzy random variables. Readers familiar with these topics can skip this section, and those interested in a detailed presentation of fuzzy theory may see Zimmermann (2001).

2.1. Fuzzy sets and fuzzy numbers

Let $\mathbf{X}$ a crisp set whose elements are denoted x. A fuzzy subset A of $\mathbf{X}$ is defined by its membership function $\mu_A : \mathbf{X} \to [0, 1]$ which associates each element x of $\mathbf{X}$ with its membership degree $\mu_A(x)$ (Zadeh, 1965). The degree of membership of an element x to a fuzzy set A is equal to 0 (respectively 1) if we want to express with certainty that x does not belong (respectively belongs) to A.

The crisp set of elements that belong to the fuzzy set A at least to the degree α is called the α-cut or α-level set and defined by:

$$A_\alpha = \{x \in X | \mu_A(x) \geq \alpha\}. \quad (1)$$

A_0 is the closure[3] of the support[4] of A.

Fuzzy numbers are numbers that have fuzzy properties, examples of which are the notions of "around ten percent" and "extremely low". Dubois and Prade (1980, p.26) characterizes the fuzzy numbers as follows

Definition 2.1. A fuzzy subset A of $\mathbb{R}$ with membership $\mu_A : \mathbb{R} \to [0, 1]$ is called a fuzzy number if

1. A is normal, i.e. $\exists x_0 \in \mathbb{R} | \mu_A(x_0) = 1$;
2. A is fuzzy convex, i.e.
$$\forall x_1, x_2 \in \mathbb{R} \mid \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}, \quad \forall \lambda \in [0, 1];$$
3. μ_A is upper semi-continuous[5];
4. supp(A) is bounded.

Definition 2.2 (*Zimmermann (2001, p. 64)*)**.** A LR-fuzzy number, denoted by $\tilde{A} = \langle l, c, r\rangle_{LR}$, where $c \in \mathbb{R}^+$ is called the central value, and $l \in \mathbb{R}^+$ and $r \in \mathbb{R}^+$ are the left and the right spread, respectively, is characterized by a membership function of the form

$$\mu_A(x) = \begin{cases} L\left(\dfrac{c-x}{l}\right) & \text{if } c - l \leq x \leq c, \\ R\left(\dfrac{x-c}{r}\right) & \text{if } r + c \geq x \geq c, \\ 0 & \text{else.} \end{cases} \quad (2)$$

$L : \mathbb{R}^+ \to [0, 1], R : \mathbb{R}^+ \to [0, 1]$ are strictly continuous decreasing functions such that $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$. L and R are called the left and the right shape functions respectively. If right and left spreads are equal and $L := R$, the LR-fuzzy number is said to be a symmetric fuzzy number and denoted $\tilde{A} = (c, \Delta)$. Δ is the spread equal to $l = r$.

For simplicity, we limit the present study to triangular[6] fuzzy numbers characterized by the shape function $R(x) := L(x) := \max\{1 - x, 0\}$. The analysis can be extended to other membership types.

Using Zadeh's extension principle (Zadeh, 1965), which is a rule providing a general method to extend a function $f : \mathbb{R}^k \to \mathbb{R}$ to the set of fuzzy numbers, we can define binary operators such as addition, subtraction, multiplication, etc, for two fuzzy numbers. When $k = 2$, this method defines the membership function of the result as follows

$$\mu_{\tilde{A}_1 \circ \tilde{A}_2}(z) = \sup_{(x_1,x_2)\in \tilde{A}_1 \times \tilde{A}_2} \left\{\min\left(\mu_{\tilde{A}_1}(x_1), \mu_{\tilde{A}_2}(x_2)\right) \mid x_1 \circ x_2 = z\right\} \quad (3)$$

where $\circ$ is the binary operator.

2.2. Fuzzy random variables

Different approaches of the concept of fuzzy random variables have been developed in the literature since the 1970s. The most often cited being introduced by Kwakernaak (1978) and enhanced by Kruse and Meyer (1987), and the one by Puri and Ralescu (1986). An extensive discussion on these two approaches is given by Shapiro (2009). For the purpose of this study, we adopt the concept of FRVs of Puri and Ralescu (1986).

Let $\mathcal{F}_c(\mathbb{R})$ denote the set of all normal convex fuzzy subsets[7] of $\mathbb{R}$ and $(\Omega, \mathcal{A}, P)$[8] a probability space.

More precisely, Puri and Ralescu (1986) have defined a FRV as follows.

3 The closure of the support of A is the smallest closed interval containing the support of A (Shapiro, 2009).

4 The support of A is the set of all x such that $\mu_A(x) > 0$ (Shapiro, 2009).

5 Semi-continuity is a weak form of continuity. Intuitively, a function f is called upper semi-continuous at point x_0 if the function's values for arguments near x_0 are either close to $f(x_0)$ or less than $f(x_0)$.

6 This assumption of simplicity is also made in numerous articles of IME such as Koissi and Shapiro (2006), Andrés-Sànchez (2007) and Berry-Stölzle et al. (2010), among others.

7 A fuzzy set $\tilde{A}$ is called a normal convex fuzzy subset of $\mathbb{R}$ if $\tilde{A}$ is normal, the α-cuts of $\tilde{A}$ are convex and compact and the support of $\tilde{A}$ is compact (Körner, 1997).

8 Where Ω is the set of all possible outcomes described by the probability space, $\mathcal{A}$ is σ-fields of subsets of Ω, and the function P defined on $\mathcal{A}$ is a probability measure.