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Optimal capital allocation in a hierarchical corporate structure[☆]

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HIGHLIGHTS

- We consider capital allocation in a hierarchical corporate structure.
- Any organizational level may have different attitudes towards risk.
- Capital allocation is considered as the solution to an optimization problem.
- An explicit unique solution to this optimization problem is given.
- The examples show the optimal allocation according to conflicting views of risk.

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ABSTRACT

We consider capital allocation in a hierarchical corporate structure where stakeholders at two organizational levels (e.g., board members vs line managers) may have conflicting objectives, preferences, and beliefs about risk. Capital allocation is considered as the solution to an optimization problem whereby a quadratic deviation measure between individual losses (at both levels) and allocated capital amounts is minimized. Thus, this paper generalizes the framework of Dhaene et al. (2012), by allowing potentially diverging risk preferences in a hierarchical structure. An explicit unique solution to this optimization problem is given. In several examples, it is shown how the optimal capital allocation achieves a compromise between conflicting views of risk within the organization.

1. Introduction

Capital allocation is an exercise whereby the total amount of economic capital available to an insurance or financial institution is apportioned to individual sub-portfolios, such as business divisions, lines of business, distinct legal entities (as in the case of an insurance group), or individual contracts. Such allocation of capital may be purely notional or involve an actual transfer of funds, depending on fungibility constraints. The purposes of capital allocation can include performance measurement, assessment of investment opportunities, portfolio management, and even incentive compensation.

While a host of capital allocation methods are described in the literature, the underlying principle is typically that the capital allocated to a particular risk should in some way reflect the contribution of that risk to the portfolio, often as captured by a risk measure. There are multiple ways of defining such contributions. Marginal cost arguments are used by Tasche (2004) in the context of performance measurement, while related game theoretical criteria emphasize principles of fairness and stability in the portfolio (Denault, 2001; Tsanakas and Barnett, 2003). Capital allocation in models with hierarchically structured dependence has been considered by Arbenz et al. (2012). Capital allocation methods derived from various notions of optimality are investigated in Dhaene et al. (2003), Laeven and Goovaerts (2004), Zaks et al. (2006), and Dhaene et al. (2012).

These last two papers are most closely related to the present contribution. In particular, Dhaene et al. (2012) formulate capital allocation as an optimization problem, where the available capital is exogenously given and the objective function is formed by summing the distances of allocated capital amounts from individual losses. Distance is measured by expected (quadratic or absolute) deviations, after a re-weighting of probabilities, which assigns higher weights to scenarios of higher relevance. It is shown that appropriate choice of such scenario weights, reflecting management

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preferences for different parts of the portfolio, can generate a wide variety of capital allocations, reproducing most of the allocation methods found in the literature. Zaks (2013) generalized such arguments to a situation case where capital is invested in a number of risky assets.

An important practical issue that is generally ignored in the above literature relates to the potentially conflicting objectives, preferences and beliefs at different levels of a financial institution's hierarchy, for example, the levels of a company board and line managers. Such conflicts may take different forms, reflecting different organizational structures and cultures. Preferences/scenario weights may be solvency-driven at the board level and price sensitive at the line-of-business level (or indeed the converse may be true). Boards may be concerned with overall portfolio performance, while line managers focus on the performance of the books they are managing. Even when preferences are consistent, the beliefs about loss probability distributions may differ, for example, reflecting the specific expertise that line managers have in relation to the liabilities they are managing. For ways in which organizational design influences the allocation of capital to competing investment projects (a problem indirectly related to what is studied here), see Stein (2002) and the references therein.

In this paper, we address the above issues by generalizing the argument of Dhaene et al. (2012) in a hierarchical setting. An augmented objective function is proposed, involving quadratic deviations between loss and allocated capital at different levels of the organization's hierarchy. There is enough flexibility in the selection of possible scenario weights, to reflect divergent risk preferences by the same stakeholder for different parts of the portfolio, as well as by different stakeholders in relation to the same part of the portfolio. Hence the conflicting views of, say, company board and line managers can be accommodated in a single framework.

An explicit unique solution to the above optimization is derived. This leads to explicit formulas for the optimally allocated capital at different (top and bottom) levels of the hierarchy. Capital allocation becomes a two-step procedure. First, capital allocated at the top level under consideration (e.g., to lines of business) is driven by a combination of preferences at both levels. Second, the allocation of those capitals at the bottom level (e.g., to individual policies) is only driven by bottom-level preferences. Thus, while board-level preferences impact on the capital available to line managers, there is no interference from above in the allocation of capital within lines of business.

Special cases where the formulas simplify are considered and a number of detailed examples are given. It is beyond the scope of this paper to complicate the rather exhaustive discussion of Dhaene et al. (2012), by considering a multitude of combinations of diverging preferences at different levels. The examples are chosen to highlight particularly pertinent cases of conflicting objectives, preferences and beliefs at different levels of a financial institution's hierarchy, as discussed above. We focus on how the optimal capital allocation derived attempts to resolve these conflicts and achieves a compromise view of risk.

The formal setting and main result are given in Section 2, while special cases and examples are discussed in Section 3. The proof of the main result is presented in Section 4. Finally, brief conclusions are given in Section 5.

2. Optimal capital allocations

2.1. Set-up

We consider a financial institution with n portfolios, where the ith portfolio is in turn divided into n_i sub-portfolios. Several situations fit this hierarchical setting, for example (i) an insurance group consisting of n legal entities, each writing n_i lines of business; (ii) an insurance company active in n lines of business, in each of which n_i (groups of similar) policies are sold; (iii) a financial institution exposed to n types of risk as defined by solvency regulation (market, credit, operational, etc.), each of which is decomposed into n_i sources of exposure.

The loss arising from the ith portfolio is denoted by the random variable X_i for $i = 1, \ldots, n$. The loss arising from the jth sub-portfolio of the ith portfolio is denoted by the random variable X_{ij} for $j = 1, \ldots, n_i$. Note that we do not in general require that $\sum_{j=1}^{n_i} X_{ij} = X_i$, though the simplifying assumption is used in the examples of Section 3. That allows for the presence of portfolio non-linearities, as well as the inclusion in X_i of deadweight costs or risks to which no capital will be allocated.[1]

We assume that an exogenously given total amount of capital K is available. This will be allocated to the n portfolios by $\boldsymbol{K} = (K_1, \ldots, K_n)$, where the *top-level capitals* add up to the total available capital, $\sum_{i=1}^{n} K_i = K$. In turn, each K_i will be allocated to n_i sub-portfolios via $\boldsymbol{k}_i = \left(k_{i1}, \ldots, k_{in_i}\right)$, where the *bottom-level capitals* add up to K_i, i.e., $\sum_{j=1}^{n_i} k_{ij} = K_i$. Denote the $\sum_{i=1}^{n} n_i$-vector of bottom-level capitals as $\boldsymbol{k} = (\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n)$.

Consistently with the arguments of Zaks et al. (2006) and Dhaene et al. (2012), the capital allocation will be derived from the general principle that *the capital allocated to a risk should be close to it, according a measure of distance that reflects management preferences*. In particular, capital allocation in this paper arises as the solution to the following optimization problem:

$$\begin{cases} \min\limits_{\boldsymbol{K},\boldsymbol{k}} & \left\{(1-\lambda)\sum_{i=1}^{n}\frac{1}{\nu_i}\mathbb{E}\left[\xi_i\left(K_i - X_i\right)^2\right]\right. \\ & \left. +\lambda\sum_{i=1}^{n}\sum_{j=1}^{n_i}\frac{1}{\nu_{ij}}\mathbb{E}\left[\xi_{ij}\left(k_{ij}-X_{ij}\right)^2\right]\right\} \\ \text{s.t.} & \sum_{i=1}^{n} K_i = K \\ & \sum_{j=1}^{n_i} k_{ij} = K_i \quad \forall i = 1, \ldots, n,\ j = 1, \ldots, n_i, \end{cases} \tag{1}$$

where the distance measures used are built with the following elements:

- A *quadratic deviation measure*, consistent with the common use of quadratic loss functions in insurance; see e.g. Lemaire (1995).
- Measures of *business volume*, $\nu_i > 0$, $\nu_{ij} > 0$ corresponding to X_i, X_{ij} respectively.
- *Scenario weights* ξ_i, ξ_{ij}, corresponding to X_i, X_{ij} respectively. Each of ξ_i, ξ_{ij} is a non-negative random variable with $\mathbb{E}[\xi_i] = \mathbb{E}[\xi_{ij}] = 1$. These weights reflect an assessment that certain scenarios (states of the world) may be more relevant as drivers of capital than others. Depending on management preferences, the variables ξ_i, ξ_{ij} may assign a higher weight on scenarios where particular (sub-)portfolios incur high losses or where market conditions are adverse. For a full discussion and several examples, see Dhaene et al. (2012). A key difference in this paper is that ξ_i, ξ_{ij} are generally *not* the same, reflecting differently defined risk preferences at different (top and bottom) levels of the organization's hierarchy.
- A constant $0 < \lambda < 1$ that reflects the balance between top-level preferences (low λ) and bottom-level preferences (high λ).

[1] For example, if capital allocation is used to derive profitability targets, there may be no allocated capital to forms of operational risk that are not directly associated with profit-making.