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Credibility theory based on trimming

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HIGHLIGHTS

- In this paper we propose a credibility theory via truncation of the loss data.
- The proposed framework contains the classical credibility theory as a special case.
- It is shown that the trimmed mean is not a coherent risk measure.
- Some related asymptotic properties are established.
- A numerical illustration is provided.

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ABSTRACT

The classical credibility theory proposed by Bühlmann has been widely used in general insurance applications. In this paper we propose a credibility theory via truncation of the loss data, or the trimmed mean. The proposed framework contains the classical credibility theory as a special case and is based on the idea of varying the trimming threshold level to investigate the sensitivity of the credibility premium. After showing that the trimmed mean is not a coherent risk measure, we investigate some related asymptotic properties of the structural parameters in credibility. Later a numerical illustration shows that the proposed credibility models can successfully capture the tail risk of the underlying loss model, thus providing a better landscape of the overall risk that insurers assume.

1. Introduction

Credibility theory allows actuaries to estimate the conditional mean loss for a given risk to establish an adequate premium to cover the insured's loss. The theory of experience-based credibility constitutes the backbone in general insurance ratemaking. Let us consider such loss random variables (rv) $X_1, \ldots, X_n$ and X from a common distribution function (df) $F(x;\theta)$ with θ representing the risk level of the insured. The losses are assumed to be independent and identically distributed conditional on $\Theta=\theta$, meaning that the risk parameter itself is a rv,[1] and $\mathbf{X}=(X_1,\ldots,X_n)$ typically stand for past experience, whereas X is the loss for the next period. Under this setup the ideal individual premium, or the hypothetical mean, for an insured with θ is simply $\mu(\theta):=E(X;\theta)$, but it is not usable because we are unable to pinpoint θ. The classical approach by Bühlmann (1967) tackles this by minimizing the expected squared loss:

$$\min_{\alpha,\beta} E\left[(\mu(\Theta)-\alpha-\beta\bar{X})^2\right],$$

where $\bar{X}=n^{-1}\sum_{i=1}^{n}X_i$ and the expectation is over all the random variables. The resulting minimizer leads to the celebrated credibility premium

$$P_c = Z\bar{X}+(1-Z)\mu, \tag{1}$$

where $\mu=E[\mu(\theta)]$ is the collective premium for the portfolio of all insureds, and

$$Z=\frac{n}{n+v/a}$$

with $v=E[\mathrm{Var}(X;\theta)]$ and $a=\mathrm{Var}[\mu(\theta)]$. While the credibility premium is seen to be an approximation to the unknown individual premium, it is actually identical to the Bayes premium $E(X|X_1,\ldots,X_n)$ for, e.g., Linear Exponential Family of distributions and its conjugate priors (Jewell, 1974). A major advantage of the credibility premium over the Bayes premium is in its ability to calculate premiums straightforwardly in non-parametric settings without specifying the underlying distributions. The reader is

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[1] We will use Θ and θ interchangeably unless it causes confusion.

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referred to standard texts by Klugman et al. (2008) and Bühlmann and Gisler (2005) for further details on the classical credibility theory.

Various adaptations and extensions have been made in the credibility literature in the last several decades following the classical Bühlmann's approach. For example, different loss functions were considered. Instead of the squared loss, the exponentially weighted loss yielding the Esscher premium principle is considered in Gerber (1980) and Pan et al. (2008). The exponential loss is considered by Ferreira (1977), Denuit and Dhaene (2001), and Morillo and Bermúdez (2003). In fact Heilmann (1989) considers various loss functions in the form $g(x)(h(x)-h(a))^2$ that contains the Esscher premium principle as a special case. In a recent article Gómez-Déniz (2008) provides a credibility premium driven from a weighted balanced loss function.

Another direction of extension is made on the probability function. Wang and Young (1998) suggests the following risk-adjusted credibility premium:

$$\int_0^\infty g[1-F(x|\mathbf{X})]dx, \tag{2}$$

where g is a distortion function (Denneberg, 1994). The risk-adjusted premium (2) reduces to the usual Bayes premium at $g(y) = y$.

In this paper we propose a credibility theory based on truncating (or trimming) the original data. More specifically the ground up losses are truncated both from above and below at different points, say, p- and q-quantiles, respectively. The idea of using truncation in credibility can be found in De Vylder (1976). However, in our approach, the target premium for which the quadratic loss is minimized is not the hypothetical mean $\mu(\theta)$, but its trimmed counterpart (3). Therefore the resulting credibility premium is different from that of De Vylder (1976) where the semilinear credibility is targeting the hypothetical mean. There are several advantages using the trimmed mean, rather than just mean.

1. With suitably selected truncation points the resulting individual premium and the credibility premium can provide a natural and intuitive basis to determine the risk loading. This can be achieved by setting $p > 0$ and $q = 1$. This choice actually gives a coherent risk measure called the Conditional Tail Expectation,[2] in the sense of Artzner et al. (1999).
2. Similarly by omitting large loss records actuaries may identify and measure the risk and impact of large claims that can have substantial impact on the classical credibility premium. This can be done buy setting $p = 0$ and $q < 1$. This is translated to the pricing in the presence of policy limits.
3. Further varying the value of (p, q), actuaries can capture the tail thickness of the underlying loss models and further examine the sensitivity of the premium to the right tail risk. For instance, two different loss models may have comparable classic credibility premium even though their tails are substantially different. The proposed model can identify and distinguish this difference.

Regarding the first item, the risk-loaded premium principle has recently received much attention in finance and actuarial communities for its connection to the discussion on risk measures. Also, there is a large literature in statistics advocating the use of trimmed mean as a robust procedure when estimating the location parameter.

The present article is organized as follows. In Section 2 basic properties of the trimmed mean are examined from a risk measure perspective. Section 3 develops some theoretical results for the proposed credibility approach. Two parametric examples are given in Section 4. In Section 5 we illustrate how the proposed approach can be used in the non-parametric setting. A numerical example is presented in Section 6, and Section 7 concludes the article.

2. Formulation

Consider a loss random variable X with df $F(x;\theta)$ and density (or probability mass function if discrete) $f(x;\theta)$. The parameter itself is assumed to be a realization of a (prior) rv Θ with density $\pi(\theta)$. If we denote the p-quantile $F^{-1}(p;\theta)$ by $Q_p(X;\theta)$, we can define the trimmed mean as

$$\mu_{p,q}(\theta) = E[X|Q_p(X;\theta) < X < Q_q(X;\theta);\theta],\quad 0 \le p < q \le 1. \tag{3}$$

Throughout the article we assume that X is continuous and that its first two moments exist to be consistent with the classical credibility theory.

2.1. Trimmed mean as a risk measure

The trimmed mean has long been studied in Statistics as a robust alternative to the mean, and the class of linear combinations of order statistics, called the L-estimator class, has been explored as an extension of the mean or trimmed mean; see, e.g., Staudte and Sheather (1990) or Hampel et al. (1986). It has also been investigated in the actuarial literature for loss model inferences; see, e.g., Brazauskas et al. (2007), Brazauskas et al. (2008), and Brazauskas et al. (2009).

Consider the following loss criterion

$$\min_d E\left[\left(\frac{I(Q_p < X < Q_q)}{q-p}\right)(X-d)^2\right], \tag{4}$$

where $I(\cdot)$ is the indicator function. The solution is, after straightforward algebra, given by the trimmed mean; the loss function in fact belongs to the class considered in the premium principle of Heilmann (1989).

The trimmed mean can be alternatively understood as a member of the Distortion Risk Measure (DRM) class. From the considerable literature on the topic of risk measures including, e.g., Artzner et al. (1999), Denuit et al. (2005), Wang (2000), it is known that a large class of risk measures, such as the Conditional Tail Expectation, Value at Risk, and Wang Transform, can be expressed as

$$\int_0^1 Q_u(X;\theta)g'(1-u)du \tag{5}$$

where $g : [0, 1] \to [0, 1]$, an increasing function with $g(0) = 0$ and $g(1) = 1$, is called the distortion function; see, e.g., Jones and Zitikis (2003) and Furman and Zitikis (2008) and the references therein for further discussions. Using a variable transformation we see that the DRM (5) can be rewritten as $E[Xg'(1-F(X;\theta))]$. This allows us to identify the DRM as the minimizer of

$$\min_a E\left[g'(1-F(X;\theta))\,(X-a)^2\right]. \tag{6}$$

The connection of the DRM and the trimmed mean is made by choosing the following distortion function for the trimmed mean

$$g(u) = \begin{cases} 0, & 0 \le u \le 1-q, \\ \dfrac{u+q-1}{q-p}, & 1-q < u \le 1-p, \\ 1, & 1-p < u \le 1. \end{cases}$$

[2] Also known as the Tail Conditional Expectation, Conditional Value-at-Risk (CVaR), and Tail Value-at-Risk (TVaR) in the literature.