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Optimal dividends with debts and nonlinear insurance risk processes

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HIGHLIGHTS

- We consider nonlinear insurance risk processes attributed to internal competition factors.
- We study optimal dividend problem with fixed transaction costs and proportional tax.
- Under some suitable hypotheses, we obtain the structure of the value function.
- Closed-form solutions to the value function are obtained in various cases.

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ABSTRACT

The optimal dividend problem is a classic problem in corporate finance though an early contribution to this problem can be traced back to the seminal work of an actuary, Bruno De Finetti, in the late 1950s. Nowadays, there is a leap of literature on the optimal dividend problem. However, most of the literature focus on linear insurance risk processes which fail to take into account some realistic features such as the nonlinear effect on the insurance risk processes. In this paper, we articulate this problem and consider an optimal dividend problem with nonlinear insurance risk processes attributed to internal competition factors. We also incorporate other important features such as the presence of debts, constraints in regular control variables, fixed transaction costs and proportional taxes. This poses some theoretical challenges as the problem becomes a nonlinear regular-impulse control problem. Under some suitable hypotheses for the value function, we obtain the structure of the value function using its properties, without guessing its structure, which is widely used in the literature. By solving the corresponding Hamilton–Jacobi–Bellman (HJB) equation, closed-form solutions to the problem are obtained in various cases.

1. Introduction

The optimal dividend problem is one of the major issues in corporate finance since its inception though an early contribution to this problem can be traced back to the seminal work by an actuary, Bruno De Finetti, in the late 1950s. The story goes back to the International Congress of Actuaries in New York in 1957, where De Finetti presented his inaugural work on a mathematical approach to an optimal dividend problem of an insurance company. The key motivation of De Finetti's study is attributed to the observation that in classical risk theory where evaluating ruin probability of an insurance company is the main concern, surplus of the company can increase indefinitely without bounds. This is, of course, not realistic. To articulate this problem, De Finetti (1957) first introduced dividend payments to the scene and considered a situation where the company wishes to maximize expectation of the present value of all dividends before possible ruin. He modeled surplus of the company as a simple random walk. Under this assumption, he obtained an elegant result that the optimal dividend-payment strategy is a barrier strategy which devises that any surplus exceeding a certain barrier level should be paid as dividends to shareholders of the company. The classic work of De Finetti (1957) has stimulated a leap of works on the optimal dividend problem. Some representative works include Bühlmann (1970), Gerber (1979), Asmussen and Taksar (1997), Taksar and Zhou (1998), Gerber and Shiu (1998, 2003), Højgaard and Taksar (2004), Øksendal and Sulem (2005), Paulsen (2007), Alvarez and Lempa (2008) and Guo and Wu (2009). Nowadays, the optimal dividend problem has become an important topic in actuarial risk theory.

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Another key topic in actuarial risk theory is the optimal reinsurance problem. Reinsurance is a major tool for insurance companies to transfer their exposures to risk to another party, namely, a reinsurer. An effective reinsurance may protect an insurance company against unexpected large losses due to insurance claims and reduce the company's earning's volatility. There is a large amount of literature on optimal reinsurance. Under the criterion of minimizing the probability of ruin of an insurance company, Schmidli (2001, 2002) studied proportional reinsurance and derived an optimal reinsurance strategy. Taksar and Markussen (2003) further studied a diffusion model with investment and proportional reinsurance. Some of the other studies on reinsurance include Taksar and Zhou (1998), Taksar (2000), Choulli et al. (2003), Højgaard and Taksar (2004), Irgens and Paulsen (2004), Choulli and Taksar (2010), Meng and Siu (2011a,b) and references therein. From a mathematical perspective, the optimal reinsurance problem can be viewed as a regular control problem, which is an important mathematical method for controlling and managing risks to which companies are exposed.

It appears that the vast literature on the optimal dividend problem and the optimal reinsurance problem mainly focus on linear insurance risk processes which may be motivated from controlling risks and dividend distributions, as well as from managing personnel (hiring/firing) policies for cooperations. Indeed, a linear insurance risk process may arise from a diffusion approximation to a compound Poisson risk process with proportional reinsurance and debt under the expected premium principle, (see, for example, Taksar and Zhou, 1998). However, as pointed out by Guo et al. (2004), the linear relationship between risk and return arising from a proportional reinsurance may be an oversimplification of realistic situations. In particular, an excessive risk exposure may not be a recipe for a high return. This motivates the quest for how an optimal reinsurance strategy may be changed when the relationship of risk and return may no longer be linear. Another motivation for nonlinear risk processes is that when the variance premium principle is considered, it was shown in Zhou and Yuen (2012) that a diffusion approximation to a compound Poisson risk process with proportional reinsurance is a nonlinear risk process with a quadratic control term in the drift coefficient. Furthermore, a nonlinear risk process may arise from the presence of internal competition factors among reinsurers. A typical situation of internal competition factors may be attributed to the effect of sentiments on risk preferences among reinsurers. In particular, when a risk-averse reinsurer has a preferred risk level and wishes to impose an additional amount of service charge on firms seeking services beyond the target level, other reinsurers may also demand extra charges for those seeking services with risk levels lower than the preferred one with a view to gaining market shares. The additional charges will enter as a quadratic control term into the drift coefficient of a diffusion-approximation risk process, and thus will lead to a nonlinear risk process. Further discussions on this point will be given in Section 2. The incorporation of nonlinear insurance risk processes is far more than a trivial issue. Indeed, it poses some theoretical challenges as the problem becomes a nonlinear regular-impulse control problem. Recently, Guo (2002) and Guo et al. (2004) introduced non-linear controlled dynamics in an optimal stochastic control problem which was motivated by a workforce control problem. They formulated the problem as a nonlinear regular-singular optimal control problem. The level of difficulty of their problem is similar to the nonlinear regular-impulse control problem.

In this paper, we consider an optimal dividend problem with nonlinear insurance risk processes, where the nonlinearity is attributed to internal competition factors of an insurance company. We also incorporate other realistic features such as constraints in regular control variables, fixed transaction costs and proportional taxes. The nonlinear regular-impulse control problem is discussed using the dynamic programming approach. By solving the Hamilton–Jacobi–Bellman (HJB) equation, closed-form solutions to the problem are obtained in various cases.

This paper is structured as follows. The next section presents a modeling framework and formulates the optimization problem. Section 3 discusses some properties and structures of the value function. In Section 4, we derive closed-form expressions for the value function and the optimal dividend strategy in each case. The final section summarizes the paper.

2. Model formulation

In this section we discuss motivation of the nonlinear risk processes to be adopted in this paper. As usual, we consider a complete, filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$, where $\mathbb{P}$ is a real-world probability and the filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfies the usual conditions, (i.e., right-continuity and $\mathbb{P}$-completeness). Let $\{W_t\}_{t\geq 0}$ be an $(\{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$-standard Brownian motion. It was shown in Taksar and Zhou (1998) that under the expected premium principle, a diffusion approximation to a compound Poisson risk process with proportional reinsurance and debt has the following form:

$$R_t = x + \int_0^t (\widetilde{\mu}U(s) - \widetilde{\delta})\mathrm{d}s + \int_0^t \widetilde{\sigma}U(s)\mathrm{d}W_s,$$

where $\widetilde{\mu}$ is the expected profit rate; $\widetilde{\sigma}$ is the volatility rate; $\widetilde{\delta}$ is the debt rate; $U(s)$ is the reinsurance policy at time s.

In this insurance risk process, the risk described by $\widetilde{\sigma}U(s)$ and the expected return $\widetilde{\mu}U(s)$ are perfectly correlated with each other. However, it was pointed out in Guo et al. (2004) that an excessive taking of risk may not necessarily be the recipe for a high expected return. This motivates the quest for a risk process beyond the class of linear risk processes. Indeed, with the consideration of internal competition factors of reinsurance markets arising from sentiment on risk preferences among reinsurers, it is of practical relevance to consider nonlinear risk processes. Particularly when a risk-averse reinsurer has a preferred risk level and wishes to impose additional service charges on insurance companies seeking services beyond the target level, other reinsurers may demand additional charges for those seeking services with risk levels below the preferred level so as to gain market shares. In this situation, it is not unreasonable to consider the following nonlinear insurance risk process:

$$\begin{aligned} R_t &= x + \int_0^t \left(\widetilde{\mu}U(s) - a(U(s)-p)^2 - \widetilde{\delta}\right)\mathrm{d}s \\ &\quad + \int_0^t \sigma U(s)\mathrm{d}W_s \\ &= x + \int_0^t (\mu U(s) - aU^2(s) - \delta)\mathrm{d}s + \int_0^t \sigma U(s)\mathrm{d}W_s. \end{aligned}$$

Here p is the preferred reinsurance level imposed by a reinsurer; a is the additional rate of charge for the deviation from the preferred level; $\mu := \widetilde{\mu} + 2ap$ and $\delta := \widetilde{\delta} + ap^2$. The nonlinear effect is described by the quadratic control term $aU^2(s)$ and a describes the nonlinear competition factor.

Furthermore, another motivation of the nonlinear risk process is the use of the variance premium principle. Indeed, it was shown in Zhou and Yuen (2012) that under the variance premium principle, the diffusion approximation to a compound Poisson risk process with proportional reinsurance and debt is given by:

$$R_t = x + \int_0^t (2\widetilde{\mu}U(s) - \widetilde{\mu}^2U^2(s) - \delta)\mathrm{d}s + \int_0^t \sigma U(s)\mathrm{d}W_s.$$