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Expected value multiobjective portfolio rebalancing model with fuzzy parameters

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ABSTRACT

In this paper we develop a multicriteria credibilistic framework for portfolio rebalancing. We use an expected value model with fuzzy parameters considering return, risk and liquidity as key financial criteria. The transaction costs are assumed to be paid on the basis of incremental discounts and are adjusted in the net return of the portfolio. A solution procedure based on fuzzy goal programming and a hybrid intelligent algorithm that combines fuzzy simulation with a real-coded genetic algorithm is presented to solve the portfolio rebalancing problem. The approach adopted here has the advantage of handling the multicriteria portfolio rebalancing problem where the fuzzy parameters are characterized by general functional forms. An empirical study is included to demonstrate the effectiveness of the solution approach and efficiency of the model in practical applications of rebalancing an existing portfolio.

1. Introduction

The field of study of portfolio selection began with the mean-variance model (Markowitz, 1952) in which return is quantified using the mean and risk using the variance. In most of the existing portfolio selection models, return and risk are used as the two fundamental criteria that govern investors' choices (Markowitz, 1952; Konno and Yamazaki, 1991; Speranza, 1993). However, it is often found that not all the relevant information for portfolio selection can be captured by these two criteria alone (Konno and Suzuki, 1995; Hallerbach and Spronk, 2002; Zopounidis and Doumpos, 2002; Steuer and Na, 2003; Ehrgott et al., 2004; Joro and Na, 2006). Other criteria might be of equal, if not greater, importance to the investor. By considering other criteria in a portfolio selection model, it may be possible to obtain portfolios in which a deficit on account of the return and/or risk is compensated by the portfolio's performance quantified on other criteria, resulting in greater overall satisfaction for the investor. Thus, multicriteria portfolio selection models have received great interest from researchers in the recent past (Fang et al., 2006; Gupta et al., 2008, 2010, 2011, 2012).

Traditionally, portfolio selection models are based on the assumption that the investor has complete information for decision making (Gupta et al., 2008, 2010, 2011). However, the information available in financial markets is often incomplete, and thus decisions are made under uncertainty. Additionally, markets are affected by vagueness and ambiguity caused by the use of expressions such as "high risk", "low profit" and "low liquidity" by the investors and the investment experts. Portfolio selection models have benefited greatly from the fuzzy set theory (Zadeh, 1965, 2005) in terms of integrating quantitative and qualitative information about the subjective preferences of the investors and expert knowledge. Assuming that the returns are fuzzy, a large body of literature is available on fuzzy mean-variance models (Zhang and Nie, 2004; Hasuike et al., 2009; Qin et al., 2009; Li et al., 2010; Zhang et al., 2010a) using possibility measure. Though the possibility measure is widely used in the literature, it has certain limitations. For example, a fuzzy event with maximum possibility value 1 may still not occur. Additionally, it is possible that two fuzzy events with different chances of occurrence may have the same possibility value; thus, the possibility value is of little information to the investor. The primary reason for these limitations is that the possibility measure is not self-dual, which is an important property both in

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theory and practice. As an alternative, Liu and Liu (2002) proposed a credibility measure that is self-dual and thus overcomes the limitations inherent in the possibility measure. A review of credibilistic portfolio selection models has appeared in Huang (2009).

Since the introduction of the mean-variance model (Markowitz, 1952), variance is widely used as a risk measure; however, it has limitations. One of the main limitations is that the variance penalizes extreme upside (gains) and downside (losses) movements from the expected return; thus, it becomes less appropriate measure of portfolio risk when probability distributions of asset returns are asymmetric (Chunhachinda et al., 1997). This is because the obtained portfolio may have a potential danger in terms of sacrificing higher expected return. In such cases, it is desirable to replace variance with a downside risk measure, i.e. a measure which only considers negative deviations from a reference return level. Semivariance is one of the best known downside risk measures, originally introduced by Markowitz (1959) and used in mean-semivariance portfolio selection models (Mao and Brewster, 1970; Markowitz et al., 1993; Rom and Ferguson, 1994; Grootveld and Hallerbach, 1999). Its advantage over variance is that semivariance does not consider values beyond the critical value (i.e. gains) as risk; thus, it is a more appropriate measure of risk when investors are concerned about portfolio under performance rather than over-performance (Markowitz et al., 1993). It may be noted that implementation of mean-semivariance portfolio selection models is, however, computationally much more tedious as compared to mean-variance portfolio selection models (Markowitz, 1959; Choobineh and Branting, 1986; Grootveld and Hallerbach, 1999). Further, there is not much research on linear (or at least linearizable) expression for semivariance that could be easily embedded in a portfolio selection model resulting in computational advantage. When membership functions of the fuzzy returns are asymmetric, fuzzy variance may also become a deficient risk measure because of the same reasons; thus, mean-semivariance models in fuzzy environment have been studied by many authors (Huang, 2008; Qin et al., 2009; Li et al., 2010; Yang et al., 2011).

Apart from return and risk, liquidity is another important parameter which can be used to measure portfolio performance. To treat uncertainty, liquidity has been considered as a fuzzy variable (Parra et al., 2001; Fang et al., 2006; Gupta et al., 2008, 2010, 2011). A detailed list of criteria which may be used for portfolio selection apart from return, risk and liquidity is given in Steuer et al. (2008).

Given an initial holding of a portfolio, at any period of time, we may consider rebalancing (adjusting) of the existing portfolio by buying and/or selling assets in response to changing market conditions. In such situations there is a transaction cost associated with buying and/or selling of an asset. Transaction cost is one of the main concerns for portfolio rebalancing that help in constructing more realistic models which incorporate market frictions (Choi et al., 2007; Kozhan and Schmid, 2009; Yu and Lee, 2011). There are studies in the literature on portfolio optimization with either fixed transaction costs (Mao, 1970; Jacob, 1974) or variable transaction costs which change as a proportion of the amount of assets traded (Morton and Pliska, 1995; Fang et al., 2006; Zhang et al., 2010a, 2011); (Zhang et al., 2010b). Usually incurred transaction costs depend on trading volume in a nonlinear way. This is because the transaction cost rate is relatively large when trading volume is small and it gradually decreases as trading volume increases.

Here, we attempt to fill some gaps of the existing literature on the proposed research topic. It may be noted that semivariance was introduced as a downside risk measure in stochastic environment; however, in fuzzy environment it is used only in a few studies (Huang, 2008; Qin et al., 2009; Li et al., 2010; Yang et al., 2011). Credibilistic framework for mean-variance portfolio selection problem was initiated by Huang (2007) and later extended to multicriteria framework by many authors (Qin et al., 2009; Li et al., 2010; Yu and Lee, 2011). To the best of our knowledge, the existing multicriteria credibilistic framework of portfolio selection/rebalancing is treated either by converting a fuzzy model into an equivalent crisp model when fuzzy parameters such as triangular and trapezoidal are used, or by considering a single-objective optimization model using all criteria except one as the constraints when fuzzy parameters are characterized by general functional forms. Furthermore, transaction costs in fuzzy environment are mainly considered either fixed costs or proportional (variable) costs (Fang et al., 2006; Zhang et al., 2010a, 2011); (Zhang et al., 2010b).

In this paper, the mean-semivariance portfolio selection model is extended to a multicriteria portfolio rebalancing model in fuzzy environment where apart from return and risk, we also consider liquidity for measuring performance of a portfolio. We propose an expected value multiobjective model with fuzzy parameters based on credibility measure of fuzzy events. Further, we consider an investment market scenario where the investor pays variable transaction costs based on incremental discount schemes whenever the amount held of any asset is changed. The transaction costs are adjusted in net return of the portfolio. For portfolio return, we consider average performance of the asset during a 36-month period. Liquidity is considered in terms of the probability of conversion of an investment into cash (turnover) without any significant loss in value (Parra et al., 2001; Fang et al., 2006; Gupta et al., 2008, 2010, 2011, 2012). The returns and liquidity are considered as fuzzy variables that may be characterized by general functional forms. Portfolio risk is calculated using the risk of the individual assets measured by using the semivariance. We combine fuzzy goal programming with a hybrid intelligent algorithm to solve the portfolio rebalancing problem. For the purpose, the worst and best solutions for each uncertain objective function are obtained using a hybrid intelligent algorithm that integrates fuzzy simulation with a real-coded genetic algorithm. The portfolio rebalancing model is then reformulated as a fuzzy goal programming model in which fuzzy goals of the objective functions are characterized by linear membership functions. We use fuzzy goal programming to achieve the highest membership value of each fuzzy goal. The desired compromise solution of the fuzzy goal programming model is also obtained using hybrid intelligent algorithm. An empirical study is presented to demonstrate effectiveness of the proposed approach.

The paper is organized as follows: in Section 2, we present some basic definitions and notation. In Section 3, we present the multiobjective expected value model of portfolio rebalancing problem in fuzzy environment. In Section 4, we present details of the hybrid intelligent algorithm and fuzzy goal programming approach to solve the model. The model is tested in Section 5 using a 36-month data series regarding 20 different assets selected from those listed on the National Stock Exchange of Mumbai, India. This section also includes a discussion of the results obtained. We conclude the paper in Section 6.

2. Preliminaries

We present some basic definitions and notation for better understanding of the paper.

Credibility theory (Liu, 2002, 2006; Liu and Liu, 2002; Li and Liu, 2006) is a branch of mathematics for studying the behavior of fuzzy events using a credibility measure. Credibility theory is based on five axioms from which a credibility measure is defined. Let Θ be a nonempty set and let $P(\Theta)$ be the power set of Θ (i.e., the collection of all subsets of Θ). Each element in $P(\Theta)$ is called an event. To present an axiomatic definition of credibility, it is necessary to assign to each event A, a number $Cr\{A\}$, which indicates the credibility that A will occur. Furthermore, to ensure that the number $Cr\{A\}$ has certain mathematical properties, the following five axioms should hold (Liu, 2006):