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Systemic risk tradeoffs and option prices

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ABSTRACT

Two new indices for financial diversity are proposed. The first is aggregative and evaluates distance from a single factor driving returns. The second evaluates how fast correlation with a stock rises as the stock falls. Both measures are here risk neutral. The CRI is also compared with coVaR. These measures are negatively related and so focus attention on different aspects of systemic risk. Unlike the coVaR focused on expected losses the CRI measures the risks of increased correlation and lack of diversity in activities. The CRI also declined consistently for AIG and LEH prior to their bankruptcies indicating that the market was active in decorrelating itself from these firms.

1. Introduction

The monitoring and measurement of systemic risk is becoming a mandatory activity of regulatory agencies. For example the Global Financial Stability Report of the International Monetary Fund Report (2009) is devoted to this issue. As a result numerous measures have already been proposed and include systemic expected shortfall proposed by Acharya et al. (2010), the conditional value at risk (coVaR) of Brunnermeier and Pedersen (2009), and a variety of econometric measures based on principal components analysis and Granger-causality tests introduced in Billio et al. (2010). Giesecke and Kim (2011) introduce a dynamic measure for systemic risk. These measures have so far been estimated from time series data. However, in contrast option prices provide an additional important forward looking data source. Option prices have also been used to measure systemic risk using them to first estimate default probabilities Capuano (2008), to infer asset return joint distributions from a contingent claims perspective (Gray and Jobst, 2009). Other risk neutral approaches include the use of data on credit default swaps Huang et al. (2011). Yet another approach employs traded vanilla options in a measure of comonotonicity as a measure of systemic risk developed in Dhaene et al. (2012) and Dhaene et al. (2012). For further details on comonotonicity we refer the reader to Dhaene et al. (2002a,b).

This growing literature is here complemented by considering more generally the measurement of systemic risk as it may prevail in any sector of the economy. The systemic risk is studied via the relationship between a sector index and its components and one may seek to measure the exposure of a component to its sector or the systemic contribution of a component to its sector via for example the exposure or contribution coVaR's introduced by Brunnermeier and Pedersen (2009). The analysis could be conducted at the level of a time series analysis using real world probabilities or risk neutral probabilities inferred from option surfaces. The focus here will be on the latter and we leave the former for a separate research study.

The objective here is to introduce and apply two new measures based on a risk neutral analysis. Furthermore estimated joint risk neutral densities are employed to compute the contributory coVaR. The first new measure is an aggregated one addressing the lack of diversity or the presence of herd behavior in sector returns. The second measures the impact of a severe down move in a component on the conditional correlation with the sector. The first measure is termed a sector diversity index (SDI) and the second is called the correlation response index (CRI). More precisely the SDI, CRI, and coVaR can be determined continuously from option data. These indices incorporate any major changes in derivative and spot markets instantaneously. They are illustrated here on market close data.

The comparison could be extended to include the marginal expected shortfall introduced Acharya et al. (2010). We did evaluate this measure along with the coVaR and found it to be highly correlated with the coVaR and report here just the coVaR. Furthermore we report the percentage drop down to the coVaR from the current index level. A $\Delta coVaR$ measures the drop from the median VaR instead and the two are related. For comparisons across names we compute a percentage relative to the current index level. For other studies comparing these measures we cite Benoit et al. (2012).

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It is observed that the CRI and coVaR are inversely related in the data, suggesting that they measure different aspects of systemic risk. The increase in both the SDI and CRI over the recent period suggests a decrease in the diversity of financial activities and hence higher correlation responses coupled with a decrease in the coVaR that could potentially be linked to the deleveraging of the financial sector. Regulatory mechanisms that reduce the coVaR yet preserve financial diversity via lowering the SDI and CRI should be the future focus of macro prudential innovation.

For the two firms going bankrupt, AIG and LEH, we observe a steady decline in the correlation response index prior to the bankruptcy. This observation suggests that the market may have been working to decorrelate itself from these firms prior to the default date. No pattern is observed with regard to coVaR in this period. The CRI may therefore provide early warnings of a pending default.

The outline of the rest of the paper is as follows. Section 2 presents the general principles underlying the construction of the aggregate index, SDI. Results on this index are provided and discussed in Section 3. Section 4 details the construction of the correlation response index, CRI. In Section 5 the CRI construction is implemented for the financial sector and the results are contrasted with coVaR. Section 6 is devoted to a study of CRI and coVaR for AIG and LEH. Section 7 concludes. All technical details are provided in the Appendix.

2. The sector diversity index

The sector diversity index is focused on estimating the extent to which asset returns in a sector are driven together. In the extreme, for example, they can be represented as driven by a single factor. In this case they become an increasing function of the movement in the common single factor. In such an extreme situation the set of returns is said to be comonotonic.

In order to measure this diversity we focus attention on a number of expectations taken with respect to the random returns. Each expectation may be seen as a valuation under a particular scenario. For a long position in the returns a conservative valuation is obtained by taking the minimum valuation over a number of scenarios. Similarly for a short position a conservative liability valuation is obtained by taking the maximum valuation over a comparable set of scenarios. The conservative asset valuation may be seen as a bid price while for a liability we get an ask price. The difference between the two is the worst case cost of entering and shortly exiting a position. Carr et al. (2011) model the capital required for a position by this magnitude and we shall here term it the required capital.

More formally the required capital computations employed here follow Carr et al. (2011). The principle underlying the capital computations is to require enough funds to cover entry and exit from a position at the worst terms when trading in a two price equilibrium economy. In these markets, people buy at ask and sell at bid prices. Such economies are described from a theoretical equilibrium perspective in Madan and Schoutens (2012) and Madan (2012). The price system delivers two prices for each random cash flow X with unfavorable purchase at a higher ask price $a(X)$ and unfavorable sale at a lower bid price $b(X)$.

Formally, there exists a set of valuation models $\mathcal{M}$ with models $Q \in \mathcal{M}$ (a potential scenario) such that

$$a(X) = \sup_{Q\in\mathcal{M}} E^Q[X],$$

$$b(X) = \inf_{Q\in\mathcal{M}} E^Q[X],$$

(Cherny and Madan, 2010). It is clear that $a(X) = -b(-X)$ and one may restrict discussion to the construction of the bid price. The concept of the ask price seen as a supremum of expected values taken over a set of potential scenarios, is also closely related to the notion of a 'coherent risk measure', see e.g. Huber (1981).

A property of such capital requirements is that the required capital of a portfolio equals the sum of the required capital over all the components when we have zero diversity and comonotonicity. More generally the capital required for a portfolio is dominated by the sum of the individual requirements. This observation provides the basic intuition behind the sector diversity index.

The capital required is then set at

$$c(X) = a(X) - b(X).$$

The ask functional is convex on the space of random variables while the bid functional is concave. Capital defined this way is then a convex functional. Hence one may compute the capital requirement on a basket (an index or an ETF) and compare it with the individual capital requirements on the underlyings. More precisely, assume we are considering a weighted basket of n components:

$$X = \sum_{i=1}^{n} w_i X_i.$$

Then we compute

$$c(X) = c\left(\sum_{i=1}^{n} w_i X_i\right) \quad \text{and} \quad \sum_{i=1}^{n} w_i c(X_i).$$

Under a perfect systemic driven market or in other words under a perfect comonotonic setting with zero diversity, one can show that both need to coincide (see for example Cherny and Madan (2009, 2010)). Under a non-perfect systemic setting however, the capital on the basket can be shown to be always less than the sum of the individual requirements: $c\left(\sum_{i=1}^{n} w_i X_i\right) < \sum_{i=1}^{n} w_i c(X_i)$. Hence, a measure of how systemic markets currently are, is the comparison of both; we compute the ratio of the basket capital requirement and the sum of the individual requirements

$$0 \le SDI = \frac{c\left(\sum_{i=1}^{n} w_i X_i\right)}{\sum_{i=1}^{n} w_i c(X_i)} \le 1.$$

A value of one then represents a fully systemic setting.

A sector diversity index (*SDI*) is obtained by taking the ratio of portfolio capital to the sum of component capitals for a portfolio representing the sector. For example one may take any sector ETF and its components to assess diversity in that sector. For an explicit construction it remains to describe the scenarios employed in the conservative valuation. The base scenario employed for expectation computations is associated with a probability consistent with the surface of option prices at the valuation time. Other prudent scenarios are derived from the base scenario by reweighting losses upwards and discounting gains. More precisely the weights employed are functions of the quantile levels of the base probability. Low quantiles associated with losses receive a high weight while high quantiles associated with gains receive a low weight.

The *SDI* is defined as a ratio with the numerator a function of the sum while the denominator is the sum of the same function applied to the components. The indices *HIX* and *CIX* introduced in Dhaene et al. (2011, 2012) are based on a similar idea using variance in the place of a coherent risk measure for the function involved.

The computations are made operational using distorted expectations and distortion functions. More precisely, when the economy sets bid and ask prices on the basis of the distribution function