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Pricing catastrophe risk bonds: A mixed approximation method

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A B S T R A C T

This paper presents a contingent claim model similar to the one described by Lee and Yu (2002) for pricing catastrophe risk bonds. First, we derive a bond pricing formula in a stochastic interest rates environment with the losses following a compound nonhomogeneous Poisson process. Furthermore, we estimate and calibrate the parameters of the pricing model using the catastrophe loss data provided by Property Claim Services (PCS) from 1985 to 2010. As no closed-form solution can be obtained, we propose a mixed approximation method to find the numerical solution for the price of catastrophe risk bonds. Finally, numerical experiments demonstrate how financial risks and catastrophic risks affect the prices of catastrophe bonds.

1. Introduction

Insured property losses from natural catastrophic risk events, such as earthquakes, hurricanes, storms, floods, tornados or man-made catastrophes, are extremely large, when compared with other types of property and casualty losses. In general, catastrophe risks are of low-loss frequency and high-loss severity. Traditionally, insurance companies hedge and transfer catastrophe risk by purchasing reinsurance contracts. However, such a reinsurance contract could be less cost-effective to the reinsurance company and may pose a severe financial stress to the reinsurance company due to the unpredictable nature of large catastrophic losses. As a result, over the last twenty years it has become increasing difficult to find a reinsurance company to cover the catastrophic losses at a reasonable cost. In order to expand the capacity of reinsurance industry, securitization of accumulated catastrophic losses in financial markets has become a timely and desirable alternative to the traditional reinsurance norm (D'Arcy and France, 1992).

Catastrophe risk bonds (CAT bonds) are one of the most important insurance-linked financial securities. The losses caused by large catastrophes could lead to a significant amount of payment for the capital market investors. The first successful CAT bond was \$85 million issued by Hanover Re in 1994 (Laster, 2001). Another CAT bond was issued by a nonfinancial firm, in 1999, which covered the earthquake losses in Tokyo for the company Oriental Land (Cummins, 2008). A \$3.4 billion risk capital was issued through 18 transactions in 2009 and the catastrophe bond market was shown to be an increasingly attractive and worthwhile supplement to the sponsor risk transfer programs (Klein, 2010).

Although there have been a number of successful issuances of the CAT bonds in recent years, few academic studies have been conducted for the pricing of CAT bonds. Cox and Pedersen (2000) evaluated catastrophe risk bonds using a representative agent technique and developed a framework of pricing CAT bonds in the incomplete market setting. Lee and Yu (2002) developed a contingent claim model that incorporated stochastic interest rates and generic loss processes with considerations of other factors, such as moral hazard, basis risk, and default risk. Lee and Yu (2007) presented a contingent-claim framework for valuing reinsurance contract that can increase the value of a reinsurance contract and reduce its default risk by issuing the CAT bonds. Egami and Young (2008) developed a method for pricing structured CAT bonds based on utility indifference pricing. Unger (2010) proposed a formulation and discretization strategy for CAT bonds model by utilizing a numerical PDE approach.

Apart from the above-mentioned studies, there also exist other articles about the pricing of the CAT bonds. Baryshnikov et al. (2001) developed an arbitrage-free solution to the pricing of the CAT bonds under the condition of continuous trading, they used compound doubly stochastic Poisson process to incorporate various characteristics of the catastrophe process. Burnecki and Kukla (2003) corrected and applied the results of Baryshnikov et al. (2001) to calculate no-arbitrage prices of a zero-coupon

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and coupon CAT bonds, and derived a pricing formula under the compound doubly stochastic Poisson model framework. Härdle and Cabrera (2010) applied the results of Burnecki and Kukla (2003) to examine the calibration of real parametric CAT bonds for earthquakes sponsored by the Mexican government. Also under an arbitrage-free framework, Vaugirard (2003a,b) adopted the jump-diffusion model of Merton (1976) to develop the first valuation model of insurance-linked securities that deal with catastrophic events and interest rate randomness. Fernández-Durán and Gergorio-Domínguez (2005) presented a methodology for the pricing of CAT bonds by considering the fact that the issuance of the CAT bond is done by the government and its main interest is to have additional funds to relieve the affected victims. Burnecki (2005) evaluated CAT bonds using a compound nonhomogeneous Poisson model with left truncated loss distribution. Jarrow (2010) developed a simple closed form solution for valuing CAT bonds, while the formula is consistent with any arbitrage-free model for the evolution of the LIBOR term structure of interest rates.

As the occurrence of catastrophe is largely unpredictable, valuing CAT bonds is very difficult. But a study of the pricing bond model plays a key role in the prevention and mitigation of natural disasters. Unfortunately, most prior studies did not take into account diverse factors that affect bond prices. In this paper, we consider a variety of factors that affect bond prices, such as loss severity distribution, claim arrival intensity, threshold level and interest rate uncertainty. Consequently, we derive a simple pricing formula for CAT bonds in a stochastic interest rates environment and show that the loss process follows a compound nonhomogeneous Poisson process.

The remainder of the paper is organized as follows. Section 2 briefly describes a CAT bond. Section 3 derives the pricing model of the CAT bonds. Section 4 conducts parameters calibration of the pricing model, and Section 5 is devoted to numerical analysis. Finally, Section 6 summarizes the paper and gives the conclusion. For ease of exposition, most proofs are presented in Appendix.

2. Brief description of a CAT bond

The simple structure of a CAT bond is presented in Fig. 1. It involves a sponsor (e.g. insurer, reinsurer, or government), who seeks to transfer the risk to investors who accept the risk for higher expected returns. The transfer of the risk to the capital market is achieved by creating a special purpose vehicle (SPV) that provides coverage to the sponsor and issues bonds to investors. The sponsor pays a premium in exchange for a pre-specified coverage if a catastrophic event of a certain magnitude takes place and investors purchase a bond. The SPV collects the capital and invests the proceeds in safe and short-term securities (e.g. Treasure bonds), which are held in a trust account. The returns generated from this trust account are usually swapped for floating returns based on the London interbank offered rate (LIBOR) that are supplied by a highly rated swap counterparty. The reason for the swap is to immunize the sponsor and the investors from interest rate (mark-to-market) risk and default risk (Cummins, 2008).

If the covered event (also called trigger event) does not happen during the term of the CAT bond, investors receive their principal plus a compensation for the catastrophic risk exposure. However, if a catastrophic risk event occurs and triggers specified in the bond contract during the risk-exposure period, then the SPV compensates the sponsor according to the CAT bond contract. This results in a partial or full principal to the investors (Loubergé et al., 1999).

Obviously, defining the default-trigger event plays an important role in structuring CAT bonds. This catastrophic event should be measurable and easily understood. In general, there are three types of triggering variables: indemnity triggers, index triggers and hybrid triggers. If the trigger event is based on the level of actual monetary losses suffered by the sponsor, then it is called an indemnity trigger. This triggering type is subject to the highest degree of the moral hazard. This phenomenon appears when the sponsor no longer tries to limit its potential losses as the risk is transferred to the investors. Therefore, moral hazard occurs due to loss control efforts by the sponsor (Lee and Yu, 2002). Although suffering from moral hazard risk, indemnity triggers eliminate basis risk by offering indemnity against modeled perils (Harrington and Niehaus, 1999).

There are three broad types of indices that can be used as CAT bond triggers: industry loss indices, modeled loss indices, and parametric indices. With industry loss indices, the payoff on the bond is triggered when estimated industry-wide losses from a catastrophic event exceed a specified threshold level. A modeled-loss index is calculated using the model provided by one of the major catastrophe-modeling firms-Applied Insurance Research Worldwide, EQECAT, or Risk Management solutions. Lastly, with a parametric trigger, the bond payoff is triggered by specified physical measures of the catastrophic events such as wind speed or the location and magnitude of an earthquake (Cummins, 2008). Index triggers help the sponsor in avoiding detailed information disclosure to the competitors, so that they can minimize the problem of the moral hazard. However, index triggers are subject to basis risk as the sponsor's losses may differ from industry losses. Here, the basis risk differs from the mismatch between the index and the sponsor's losses. Therefore, hybrid triggers can be resolved between the moral hazard and the basis risk. For example, under both index- and parametric-based triggers, the sponsor is limited to no capability in over-statistic the losses (Cummins et al., 2004; Damnjanovic et al., 2010).

3. Valuation framework

3.1. Modeling assumptions

Let $(\Omega, \mathcal{F}, \mathcal{P})$ denote a probability space, where Ω is the set of states of the world, $\mathcal{F}$ is a σ-algebra of subsets of Ω, and $\mathcal{P}$ is a probability measure on $\mathcal{F}$. Continuous trading interval $[0, T]$ for a fixed $T > 0$. An increasing filtration $\mathcal{F}_t \subset \mathcal{F}, t \in [0, T]$. Let $\{V_t : t \in [0, T]\}$ denote the CAT bond price process for all $T \in [0, T]$, which is modeled by many factors: type of region, kind of loss event, sort of insured property, and interest rates uncertainty, etc. Let $\{L_t : t \in [0, T]\}$ denote the aggregate loss process; $\{\mathcal{N}_t : t \in [0, T]\}$ is a (non)homogeneous Poisson process with an intensity parameter λ_t; $\{X_j : j \geq 1\}$ is a sequence of independent and identically distributed (i.i.d.) random variables. In addition, let $\{r_t : t \in [0, T]\}$ denote the spot interest rate process (or the force of interest). $\{\mathcal{W}_t : t \in [0, T]\}$ is a standard Brownian motion and accounts for the uncertainty of interest rates.

3.2. Valuation theory

In an arbitrage-free opportunities financial market, the value of the contingent claim $\{\mathcal{C}_T : T > t\}$ at time t can be expressed as

$$V_t = E_t^Q (D(t, T)\mathcal{C}_T | \mathcal{F}_t), \tag{1}$$

where E_t^Q denotes expectation under an equivalent martingale measure (often called the risk-neutral pricing measure), given $\mathcal{F}_t$. $D(t, T) = \exp(-\int_t^T r(s)ds)$ is a stochastic discount factor. This expression is very general, and it can be stated that in the absence of an arbitrage opportunities financial market, there exists a stochastic process $D(t, T)$ that prices the contingent claim $\mathcal{C}_T$.