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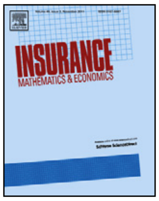

Best portfolio insurance for long-term investment strategies in realistic conditions

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ABSTRACT

Constant proportion portfolio insurance (CPPI) strategies implemented in continuous time on asset prices following geometric Brownian processes are expected utility maximising for investors with HARA utilities. But, in reality, these strategies are implemented in discrete time and asset prices might jump. We show that under these more realistic circumstances, optimal CPPI strategies are still superior to optimal option based portfolio insurance (OBPI) strategies. The effects of discrete replication and jumps on optimal strategy parameters and certainty equivalent returns (CER) are examined by simulation and turn out to be minor in typical circumstances. Hence the much discussed gap risks are unimportant for investors in both portfolio insurance strategies and comparable for insurers of the gap risks.

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1. Introduction

Portfolio insurance strategies are designed to limit downside risk by ensuring a predefined floor whilst allowing participation in the upside potential of a risky asset. Their popularity is increasing amongst investors, e.g. pension funds, that seek insurance not only against abrupt falls in the markets, such as the crash in equities after the default of Lehman Brothers, but also against general downturns such as following the collapse of the dot.com bubble in the early 2000s and the 2007–08 subprime crisis. They also attract investors which otherwise would not consider investing in the riskier asset classes such as equities and commodities. This paper examines which of two approaches to portfolio insurance is preferable.

The two most popular strategies are option based portfolio insurance (OBPI) and constant proportion portfolio insurance (CPPI). OBPI strategies, that protect an investment with a put option on the risky asset, were first discussed by Leland and Rubinstein (1976). Alternatively, one can secure a floor with an investment in a risk-free asset (a bond or a savings account) and the purchase of a call option on the risky asset. OBPI is a static method if the option can be bought, but in practice the option often needs to be replicated using a dynamic, discretely monitored investment strategy.

Merton (1971) and Brennan and Solanki (1981) derive the optimal investment payoff for a HARA utility investor in a Black–Scholes economy with a risky and a risk-free asset. They show that when the risky asset follows a geometric Brownian process the optimal payoff consists of an investment floor plus a power on the underlying risky asset price. Perold (1986) and Black and Jones (1987) introduce the concept of CPPI strategies; Perold and Sharpe (1988) analyse its properties further and show that, in continuous time, CPPI strategies replicate the optimal payoff for HARA investors. A CPPI strategy ensures a predefined floor by dynamically rebalancing allocations between the risky asset and a risk-free asset. A constant proportion or multiplier, m, of the excess value of the investment above the floor (the

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buffer) is allocated to the risky asset, the rest is invested risk-free. The floor and the multiplier are exogenous variables to the model and are determined by the investor's risk attitude and the investor's views on the risky asset price dynamics. The lower the floor and the higher the multiplier, the greater the allocation to the risky asset. The investor then has a higher upside potential but the floor is approached more quickly if the risky asset price falls.

As OBPI and CPPI strategies offer alternative downside protection, it is natural to examine under what circumstances an investor should prefer one type of protection to the other. Zhu and Kavee (1988) use Monte Carlo simulation to compare various sample statistics of replicated OBPI and CPPI payoffs. El Karoui et al. (2005) prove a very general result: for any concave utility function and in a complete market, the investment strategy that maximises expected utility (EU) subject to providing a downside protection is the unconstrained optimal strategy with a put option written on it, struck at the desired downside protection level. This result holds for both European and American style downside protection. Of course, the optimal unconstrained strategy is usually a dynamic rather than a buy and hold strategy, for example, a constant mix strategy in the case of CRRA utilities and geometric Brownian price dynamics. Bertrand and Prigent (2005), Annaert et al. (2009), and Zagst and Kraus (2011) compare OBPI and CPPI using stochastic dominance criteria. Bertrand and Prigent (2005) assume the geometric Brownian case and conclude that there is no evidence of strong or weak stochastic dominance between the two strategies, but one strategy may dominate the other in a mean-variance sense, depending on the value of the CPPI multiplier. Zagst and Kraus (2011) extend this analysis to considering second and third order stochastic dominance, deriving conditions for the strategy parameters and market parametrisations such that CPPI stochastically dominates OBPI to the second order at maturity. Annaert et al. (2009) simulate from an empirical distribution and also find no stochastic dominance between OBPI and CPPI. Furthermore, they consider a broader range of performance measures without finding that one strategy type systematically outperforms the other. Bertrand and Prigent (2011) show the dominance of CPPI strategies under Kappa performance measures by comparing theoretical payoffs in the geometric Brownian case and when the risky asset follows a jump process with Poisson distributed jump term.[1]

When the risky asset price follows a geometric Brownian diffusion process the portfolio value of a continuous time CPPI strategy can, theoretically, never reach the floor. But there is ample evidence for the existence of asset price jumps, if only because most markets trade during limited time periods every day and there are gaps between closing and opening prices. Hence in reality portfolio insurance strategy cannot be implemented in continuous time. Cont and Tankov (2009) examine the gap risk – the risk of falling below the floor – and derive the gap loss distribution and various associated risk measures in the context of a jump-diffusion price process. De Franco and Tankov (2011) maximise the investor's utility when gap risk is covered by a third party. Zhu and Kavee (1988) compare various sample statistics of simulated CPPI and OBPI portfolios and Bertrand and Prigent (2011) show the outperformance of CPPI strategies over OBPI under the Omega measure, both, for a risky asset following a compound Poisson process. Table 1 gives an overview of the most relevant papers in the portfolio insurance literature and briefly summarises their contributions.

There are different approaches to the modelling of discontinuous returns. Widely used is the model by Merton (1976) who adds a Poisson-driven jump term to a standard geometric Brownian process to account for rare sudden moves. As an alternative, Madan and Seneta (1987) introduce time-changed Lévy processes to model long-tailed stock return distributions. The authors consider pure jump processes that allow small moves to occur with a higher probability than large moves. This is a generalisation of the results of Clark (1973) who introduces subordinated processes that make use of a random time-change in a geometric Brownian process. The time-jump and the Brownian process are taken to be independent. The price can jump upwards or downwards, but the geometric process prohibits negative values for the risky asset after the occurrence of a jump. Empirical research supports the time-change modelling of asset returns. For instance Geman and Ané (1996) show that calendar-time returns are not normally distributed, as often assumed, but that returns on a unit trade basis follow a normal distribution. Madan et al. (1998) confirm that a Brownian motion time-changed process that models time with gamma distributed jumps fits historical returns significantly better than standard diffusion models.

Our analysis extends previous research as follows. First, we use a certainty equivalent return (CER) to compare the performance of CPPI and OBPI strategies in realistic circumstances. We base this CER on a two-parameter HARA utility function, which encompasses most common types of utility functions, thus representing investors with very diverse risk preferences. Second, we argue that comparing a static OBPI strategy with a fixed payoff function to a dynamic CPPI strategy with replication errors would be unfair. The CPPI payoff profile under continuous replication and a geometric Brownian price process has a fair, path-independent price that can be calculated as for standard options. We can therefore compare the two defined payoffs based on their fair prices in a complete market. Third, we recognise that standard options for portfolio insurance are not always available. Thus we compare the performances of the two strategies when implemented with delta replication in discrete time. Fourth, we search for the optimal payoff under HARA utilities in a market with discontinuous returns modelled via a time-changed geometric Brownian process.

In Section 2 we specify the continuous and discontinuous price dynamics for a risky asset price. We introduce portfolio insurance strategies in Section 3, show the optimal payoff under continuous returns and derive an approximation for the optimal payoff profile under discontinuous returns. Section 4 introduces a calibration method for a simple time-changed Brownian process and illustrates its application on an equity index and a single stock price. Despite the simplicity of this process we show that it fits empirical returns better than a geometric Brownian process. Section 5 discusses the results for different time horizons and investors with diverse sensitivities of risk tolerance to wealth. We conclude and comment on the merits of CPPI strategies in Section 6.

2. Model specification

In a Black–Scholes economy the risky asset price follows a standard geometric Brownian diffusion process,

$$S^c(t) = S(0)\ \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\ t + \sigma\ W(t)\right), \tag{1}$$

with initial risky asset price $S(0)$, drift μ, constant diffusion coefficient σ and a Brownian motion $W(t)$ at any time t. Whilst the drift and volatility are assumed constant, the analysis could be easily extended to deterministic drift $\mu(t)$ and volatility $\sigma(t)$. The

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[1] Other studies on portfolio insurance strategies include Brennan and Schwartz (1988), Black and Rouhani (1989), Black and Perold (1992), Bookstaber and Langsam (2000), Cesari and Cremonini (2003) and Bertrand and Prigent (2003).

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