Insurance: Mathematics and Economics 52 (2013) 286–299

Contents lists available at SciVerse ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Pricing inflation products with stochastic volatility and stochastic interest rates

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ARTICLE INFO

Article history:
Received June 2011
Received in revised form September 2012
Accepted 8 January 2013

JEL classification:
C02
C13
C58
C63
G12
G13
G22
G23

Keywords:
Heston Hull–White model
Inflation
Affine diffusion processes
Monte Carlo simulation
Indexation provision
Pension fund

ABSTRACT

We consider a Heston type inflation model in combination with a Hull–White model for nominal and real interest rates, in which all the correlations can be non-zero. Due to the presence of the Heston dynamics our derived inflation model is able to capture the implied volatility skew/smile, which is present in the inflation option market data. We derive an efficient approximate semi-closed pricing formula for two types of inflation dependent options: index and year-on-year inflation options. The derived pricing formulas allow for an efficient calibration of the inflation model. We also illustrate our approach using a real-life pension fund example, where the Heston Hull–White model is used to determine the value of conditional future indexations.

1. Introduction

Inflation-dependent derivatives are increasingly important in financial engineering. As a consequence, inflation[1] markets are becoming more active, liquid and transparent. Broker volumes increased substantially from late-2002, driven by a rise in the need to hedge, for example, retail products. Inflation derivatives have been traded for over a decade starting in the UK in the early 1990s. Since 2000, the market for inflation derivatives has seen a rapid growth in volumes and in types of products across various markets and linked to various domestic and regional inflation indices, such as, French CPI, Eurozone HICP, US CPI, etc. (see Fig. 1.1).

Many pension funds, (life) insurance companies[2] and banks trade these inflation-dependent derivatives. Pension funds are, for example, interested in the conditional future indexation of pension rights, which can be viewed as an exotic derivative depending on the CPI.

Modeling derivative products in finance often starts with the specification of a system of stochastic differential equations (SDEs). Such a SDE system consists of economic state variables like stock

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1 Inflation is defined as a rise in the general level of prices of goods and services in an economy over a certain period of time (usually one year). The price level is usually measured by a so-called Consumer Price Index (CPI), which reflects the actual price level of a basket of typical consumer goods. The inflation rate is then defined as the percentage change of the CPI.

2 For (life) insurance companies it is important, due to (among others) regulation and new accounting standards, to value their liabilities, which contain so-called (inflation dependent) 'embedded options', as market consistent as possible. Embedded options are rights in insurance policies or pension contracts that can provide a profit to policy holders but never a loss (see for more information, for example, van Bragt and Steehouwer, 2007).

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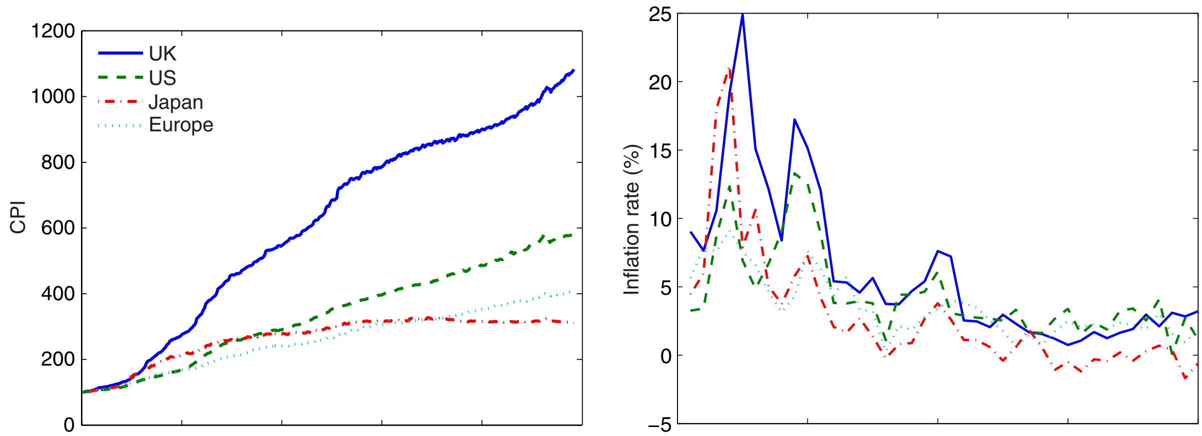
doi:10.1016/j.insmatheco.2013.01.003



(a) CPIs (monthly data (31/01/1970–31/12/2010)).

(b) Inflation rates (yearly data (31/12/1971–31/12/2010)).

Fig. 1.1. Historical overview of CPIs and inflation rates.

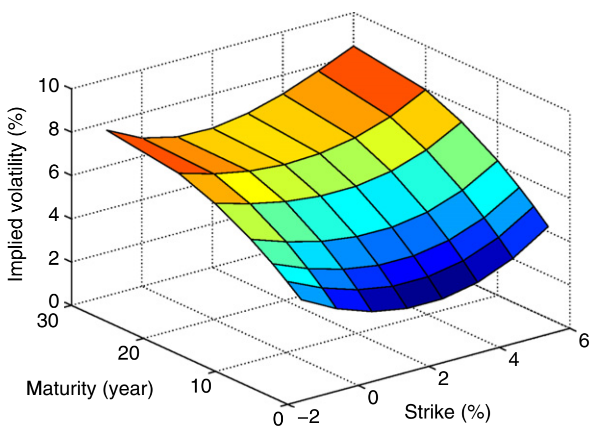


Fig. 1.2. Market implied volatilities of (Euro) inflation indexed options as of September 30, 2010.

prices, inflation, nominal and real interest rates and volatility. By imposing a correlation structure (between the Brownian motions) on this system of SDEs one can define so-called hybrid models, and use them for pricing exotic derivatives, see, for example, Grzelak and Oosterlee (2011), Grzelak and Oosterlee (2010) or van Haastrecht and Pelsser (2011).

The well-known Fisher (1930) equation defines a relation between the nominal and real interest rates on the market and the break-even inflation rate.[3] Therefore, the use of stochastic nominal and real interest rates is crucial for an accurate inflation pricing model. Furthermore, as it turns out, according to Kruse (2007), there is a significant skew/smile present in the inflation option market data in the sense that the implied Black–Scholes (BS) volatilities are not constant for different strike levels and maturities (like in the stock or currency option markets). In Fig. 1.2 the market implied volatility smile is clearly visible.

Because of this smile/skew effect in the inflation option market data, the Heston (1993) model is often used in practice, as this model is capable of capturing this effect.[4] The variance process of the CPI is then modeled by a so-called Cox–Ingersoll–Ross (CIR) process (see Cox et al., 1985). Recently, much attention has also been devoted in the literature to stochastic volatility driven by a Schöbel–Zhu process (see for example van Haastrecht and Pelsser, 2011) in combination with stochastic interest rates to model the CPI. In van Haastrecht and Pelsser (2011) also a special case of the Heston model in combination with stochastic interest rates was investigated, where some correlations were assumed to be zero. However, the case of a full correlation structure is of particular interest in this article.[5]

In this article we model the CPI by the Heston model, coupled with stochastic nominal and real interest rate processes that are driven by the one-factor Hull–White model.[6] Our focus is on the fast valuation of inflation index cap/floor options and year-on-year (YoY) inflation cap/floor options,[7] because for these products the speed of valuation is crucial for calibration. We derive an efficient pricing engine for these options, so that calibration of our inflation model can be done relatively fast. The key to obtaining the pricing formulas is the derivation of the discounted log-CPI characteristic function (ChF) under the T-forward measure. Since the ChF to be derived contains expressions which have to be evaluated numerically, efficient numerical techniques are developed as well.

This paper is organized as follows. In Section 2 we discuss the coupled inflation-interest rate model and derive the model under the T-forward measure. In Section 3 we discuss the valuation of two inflation-dependent options: inflation index caps/floors and YoY inflation caps/floors. In Section 4 we present numerical results, which include calibration results. We also devote attention to the comparison between the Heston and the Schöbel–Zhu model. In Section 5 we illustrate our approach using a real-life pension fund example, where the Heston Hull–White model is used to determine the value of conditional future indexations. We conclude in Section 6.

2. Specification of the inflation model

We consider the Heston model in which interest rates are modeled by the one-factor Hull–White interest rate model (see Brigo and Mercurio, 2006, pp. 71–80) to model the CPI. We call this inflation model the Heston Hull–White inflation (HHWi) model.

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3 The break-even inflation rate is the yield spread between nominal and inflation-linked bonds and is a fundamental indicator of inflation expectations.

4 The Heston model is for example well established for pricing stock and currency derivatives, however, not yet for pricing inflation derivatives.

5 It turns out that these correlation parameters can be influential when pricing exotic derivatives.

6 A Hull–White model is a special case of a (multi-factor) Gaussian model (see Brigo and Mercurio, 2006, Chap. 3 and 4).

7 YoY cap/floor options are defined as a series of forward starting call/put options.