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Common mortality modeling and coherent forecasts. An empirical analysis of worldwide mortality data

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ABSTRACT

A new common mortality modeling structure is presented for analyzing mortality dynamics for a pool of countries, under the framework of generalized linear models (GLM). The countries are first classified by fuzzy c-means cluster analysis in order to construct the *common sparse age-period model* structure for the mortality experience. Next, we propose a method to create the *common sex difference age-period model* structure and then use this to produce the *residual age-period*model structure for each country and sex. The time related principal components are extrapolated using dynamic linear regression (DLR) models and coherent mortality forecasts are investigated. We make use of mortality data from the "Human Mortality Database".

1. Introduction

The aim of this paper is to construct common mortality trends from a pool of countries with similar mortality experiences, and accordingly to model and forecast mortality rates for each individual country in a coherent manner, allowing for age-period effects. Many authors have discussed the benefits of common mortality modeling and coherent forecasts. Li and Lee (2005) report that mortality patterns and trajectories in closely related populations are likely to be similar in some respects, and differences are unlikely to increase in the long run, and it should therefore be possible to improve the mortality forecasts for individual countries by taking into account the patterns in a larger group. It is apparent that the populations of the world are becoming more closely linked by communication, transportation, trade, technology, and disease. They state that it would be improper to prepare mortality forecasts for individual national populations in isolation from one another and still more so for the regions within a country. They further argue that it seems likely that forecasts for individual countries could be improved by exploiting the additional information contained in the experience of similar countries. Non-divergent forecasts for sub-populations within a larger population have been labeled "coherent". Lee (2003) reports that national mortality trends should be viewed in a larger international context rather than being analyzed and projected individually, and the approach of forecasting mortality for individual countries with reference to the international context is very appealing, and should be the natural route for future developments to take. Also, he believes that whether this approach is applied to life expectancy itself, or to a Lee–Carter type k factor, or in some other way, will have to be settled by further research. Wilson (2001) has documented a global convergence in mortality levels, and states that, if we consider demographic transition in the light of a broader modernization theory, it is clear that social and demographic change has progressed far more rapidly than economic development, and that a large majority of the world's population is (or soon will be) demographically "modern" by any definition. White (2002) has reported convergence in life expectancy among 21 industrialized countries during the postwar period, and he mentions that the wealthy world is merely becoming more similar in its lifestyles, and globalization may be occurring among rich countries in practices affecting mortality, and this could lead to converging mortality patterns. Further, many authors discuss the processes of catch-up and convergence. Oeppen and Vaupel (2002) have noted that some countries converge towards the leader (e.g. Japan), some have moved away from it (e.g. the US in recent decades), and some move more or less parallel to it. White (2002) has found that nations experience more rapid life expectancy gains when they are farther

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below the international average, and conversely, and therefore tend to converge towards the average.

The remainder of the paper is organized as follows. In Section 2, utilizing mortality data from the "Human Mortality Database, we group 35 countries using fuzzy c-means cluster analysis. In this way, we form the West-cluster and East-cluster countries, for both sexes, and we construct the common age–time mortality dynamics, using 19 males West-cluster countries. Next, we employ Sparse Principal Component Analysis (SPCA) to these common mortality rates, in order to derive the *common age-period model* structure, for both sexes. In Section 3, we construct the sex difference mortality dynamics, and compare the mortality experience of males and females. In Section 4, we analyze the residual particularities for each country, based on the *common age-period model* for males, and the *common age-period* and *sex difference model* structure for females. In Section 5, we utilize dynamic linear regression (DLR) model structures, to implement coherent forecasts. Finally, in Section 6, we discuss the results and offer some concluding comments.

2. Common age-period mortality dynamics

Initially, we need to choose which countries will be selected to describe the common patterns of mortality dynamics. We require the pooled countries to contain similar characteristics. Cluster analysis is a reasonable approach to separate the countries into clusters with similar mortality dynamics. Data clustering is the process of dividing data elements into classes or clusters so that items in the same class are as similar as possible, and items in different classes are as dissimilar as possible.

Following Hatzopoulos and Haberman (2009), the log-graduated central mortality rates, in age-period effects, can be decomposed as an *age-period association model* structure (see model 2, Hatzopoulos and Haberman, 2009). According to this model structure, the main time effects component, $b(t)$ values, is an index of the level of mortality that captures the overall time trend at all ages, and summarizes the overall mortality dynamics across time. Also, the main time trends are independent of the level of mortality (a zero centered vector, constructed by linear combinations of principal component scores). Therefore, we could base our cluster analysis on these main time trends. In classic cluster analysis, the data are divided into distinct clusters, where each data element belongs to exactly one cluster. Alternatively, in fuzzy clustering, data elements can belong to more than one cluster, each associated with a membership level. This indicates the strength of the association between that data element and a particular cluster. Utilizing Fuzzy C-Means cluster analysis on the main time effects, for each country, we can distinguish the similarities or the dissimilarities among the countries. One of the most widely used fuzzy clustering algorithms is the Fuzzy C-Means (FCM) Algorithm. This technique was originally introduced by Jim Bezdek as an improvement on earlier clustering methods (Bezdek, 1981). The FCM algorithm attempts to partition a finite collection of c elements $X = X_1, X_2, \dots, X_c$ into a collection of f fuzzy clusters. The algorithm returns a list of f *cluster* multidimensional *centers* $C = C_1, C_2, \dots, C_f$, and a partition matrix of *membership grades* $U = u_{ij} \in [0, 1]$, $i = 1, \dots, c, j = 1, \dots, f$, where each element u_{ij} tells the degree to which element X_i belongs to cluster C_j.

We utilize the mortality data from the "Human Mortality Database" (HMD) (www.mortality.org). The bulk of countries (35 countries), from the HMD, have a common time range 1960–2006, and hence we apply the FCM algorithm to those national mortality data. Experiments with different numbers of clusters give a distinctive separation with $f = 2$ clusters. In Table 1, the k_r columns give the optimum number of GLM parameters for the *graduated (or full) model* structure, according to the Bayesian Information Criterion (BIC) (see Hatzopoulos and Haberman, 2009, Eq. 2.1). The μ_r-values are the overall means for each country, according to the *age-period association model* structure, and equal to the average value of the logged central rates of mortality across age and time. The u_{i1}-values give the fuzzy c-means membership levels for the first cluster (the membership value for the second cluster is complementary to the first one).

According to the u_{i1}-values, in Table 1, we form the males and females first cluster countries, which consists of 25 countries and 26 countries respectively. The common countries that belong to the first cluster, for both sexes, are the following 24 countries: Sweden, France, Denmark, E&W (England and Wales), Norway, Netherlands, Scotland, Italy, Switzerland, West Germany, Finland, Spain, Ireland, Belgium, Austria, Portugal, Luxembourg, Australia, New Zealand, Canada, USA, Japan, Czech Republic and East Germany. The countries are sorted in decreasing manner according to their fuzzy c-means u_{i1}-membership levels. White (2002) has observed that a group of 17 countries with a real GNP per capita of at least half that of the United States in 1970 and continuously democratic government, show strong convergence on life expectancies, for the period 1955–96. The countries involved are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, West Germany, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, United Kingdom and USA. We note that all of these 17 countries belong to the first West-cluster (Table 1, u_{i1} values). Also, Wilson (2001), using data from the *World Population Prospects* (United Nations 1999), has discussed the fundamental differences in the respective histories of the First, Second, and Third Worlds over the last 50 years which justify the trichotomy: developed, developing and former Communist states of the Soviet Union and Eastern Europe. In that assessment, "former Communist" is defined as the Soviet Union and all European countries ruled by Communist parties in the Cold War era, and the "developed" category includes the other European countries, along with the United States, Canada, Australia, New Zealand, and Japan, and "developing" is the rest of the world. In Table 1, all of the countries with large u_{i1} values belong to the "developed" countries, except for the Czech Republic and East Germany.

In addition, applying the FCM clustering algorithm to the remaining East-cluster countries (10 countries for males and 9 for females), gives another distinct pair of countries. The v_{i1}-values, in Table 1, give their fuzzy c-means membership levels for the first East sub-cluster. We observe distinct v_{i1}-values for the countries: Belarus, Ukraine and Russia, for both sexes, which form the first sub-cluster of the East-countries.

In panel 1.M.a of Fig. 1, we display the fuzzy cluster centers, (C_1, C_2), in time effects for the males West-cluster countries (graph with decreasing trend) and the males East-cluster countries (graph with relative steady trend). Panel 1.F.a, similarly displays the cluster centers for the females West-cluster countries (graph with decreasing trend) and the females East-cluster countries (graph with slight decreasing trend). In panels 1.M.b and 1.F.b, respectively, we display the cluster centers, (C_1, C_2), for the two East sub-cluster countries for both sexes. We note the change in the trend of the cluster centers after 1990s, for each East sub-cluster, for both sexes. The increasing trend, particularly after the 1990s, for males refers mainly to Belarus, Lithuania, Ukraine and Russia and for females refers to Belarus, Ukraine and Russia (with larger v_{i1} values, Table 1). For the remaining East-cluster countries, namely: Poland, Slovakia, Hungary, Estonia, Latvia and Bulgaria for males, and Hungary, Iceland, Estonia, Bulgaria, Latvia and Lithuania for females (with smaller v_{i1} values, Table 1), the decreasing trend is noticeable after the 1990s, for both sexes.

Although we could employ the West-cluster mortality experience for the time period 1960–2006 for fitting the models, we need