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Fitting insurance claims to skewed distributions: Are the skew-normal and skew-student good models?

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ABSTRACT

This paper analyzes whether the skew-normal and skew-student distributions recently discussed in the finance literature are reasonable models for describing claims in property-liability insurance. We consider two well-known datasets from actuarial science and fit a number of parametric distributions to these data. Also the non-parametric transformation kernel approach is considered as a benchmark model. We find that the skew-normal and skew-student are reasonably competitive compared to other models in the literature when describing insurance data. In addition to goodness-of-fit tests, tail risk measures such as value at risk and tail value at risk are estimated for the datasets under consideration.

1. Introduction

The normal distribution is the most popular distribution used for modeling in economics and finance. In general, however, insurance risks have skewed distributions, which is why in many cases the normal distribution is not an appropriate model for insurance risks or losses (see, e.g., Lane, 2000; Vernic, 2006). Besides skewness, some insurance risks (especially those exposed to catastrophes) also exhibit extreme tails (see Embrechts et al., 2002). The skew-normal distribution as well as other distributions from the skew-elliptical class thus might be promising alternatives to the normal distribution since they preserve advantages of the normal one with the additional benefit of flexibility with regard to skewness (e.g., with the skew-normal) and kurtosis (e.g., with the skew-student).

In this paper, we analyze whether these skewed distributions are reasonably good models for describing insurance claims. We consider two datasets widely used in literature and fit the skew-normal and skew-student to these data. A number of benchmark models are involved in the model comparison, as well as a goodness-of-fit procedure, in order to compare the performance of the skewed distributions in describing the insurance claims data. The motivation for consideration of the skew-normal and the skew-student is that these are popular in recent finance literature (Adcock, 2007, 2010; De Luca et al., 2006), easy to interpret, and easy to implement.

This work is related to Bolancé et al. (2008), who fit the skew-normal and log skew-normal to a set of bivariate claims data from the Spanish motor insurance industry. To our knowledge, Bolancé et al. (2008) is the only paper to date that uses the skew-normal distribution to fit insurance claims. We build on and extend their results by considering the skew-student distribution and by using different datasets. Furthermore, our analysis is broader than Bolancé et al. (2008) in that we compare our results to a large number of – 18 – alternative distributions, whereas Bolancé et al. (2008) restrict their presentation to the normal, the skew-normal, and a transformation kernel approach. We also include the transformation kernel approach as a non-parametric alternative in our discussion.

To preview our main results, we find that the skew-student and skew-normal are reasonably good models compared to other models presented in literature (see, e.g., Kaas et al., 2009). Given that this paper presents only some first tests of claims modeling in actuarial science using two well-known datasets, we call for more applications of these distributions in the field of insurance in order to more closely analyze whether the skew-normal and skew-student are promising distributions for claims modeling.

The remainder of the paper is organized as follows. In Section 2, we describe the methods we use in our estimation, especially the models involved. Section 3 presents the data. The estimation results are set out in Section 4, both those pertaining to goodness of fit and risk measurement. Conclusions are drawn in Section 5.

2. Skew-elliptical distributions

We briefly describe the distributions to be investigated in this paper, along with a short description of the benchmark models we use in the goodness-of-fit context. More detail on skewed distributions can be found in Genton (2004) and a fuller description

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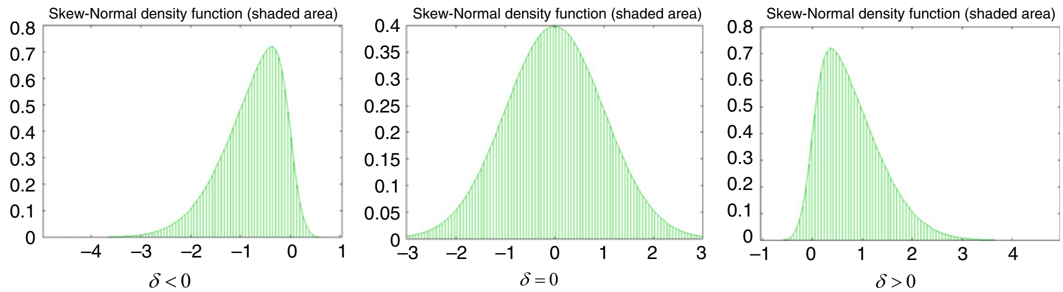

Fig. 1. Skew-normal distribution for three different values of delta.

of the benchmark models can be found in actuarial textbooks, for example, Mack (2002) or Panjer (2007).

2.1. Skew-normal

A continuous random variable X has a skew-normal distribution if its probability density function (pdf) has the form:

$$f(x) = 2\phi(x)\Phi(\alpha x). \tag{1}$$

α is a real number, $\phi(\cdot)$ denotes the standard normal density function, and $\Phi(\cdot)$ its distribution function (see Azzalini, 1985). The distribution shown by Eq. (1) is called the skew-normal distribution with shape parameter α, i.e., $X \sim SN(0, 1, \alpha)$. The skew-normal distribution reduces to the standard normal distribution when $\alpha = 0$ and to the half-normal when $\alpha \to \pm\infty$. In both empirical and theoretical work, location and scale parameters are necessary. These can be included via the linear transformation $Y = \xi + \omega X$, which is said to have the skew-normal distribution $Y \sim SN(\xi, \omega^2, \alpha)$, with $\omega > 0$. The parameters ξ, ω, and α are called location, scale, and shape, respectively. When $\alpha = 0$, the random variable Y is distributed as $N(\xi, \omega^2)$.

An alternative representation of the skew-normal that is especially popular in financial modeling is the characterization of skew-normality given by Pourahmadi (2007). A continuous random variable $Y \sim SN(\xi, \omega^2, \alpha)$ can be written as a special weighted average of a standard normal variable and a half-normal one. Y is said to have a skew-normal distribution if and only if the following representation holds:

$$Y = \xi + \omega X = \xi + \omega\left(\delta\,|Z_1| + \sqrt{1-\delta^2}Z_2\right), \tag{2}$$

with $\delta = \alpha/\sqrt{1+\alpha^2} \in [-1, 1]$. Z_1 and Z_2 are independent $N(0; 1)$ random variables. Y collapses into $N(\xi, \omega^2)$ if $\delta = 0$. Eq. (2) offers a direct financial interpretation, i.e., besides the location parameter ξ, the return Y is driven by two components:

- a half-Gaussian driver $|Z_1|$ modulated by $\omega\delta$, and
- a Gaussian driver Z_2 modulated by $\omega\sqrt{1-\delta^2}$.

The parameter δ plays a key role in determining the skewness since δ weights the presence of a half–Gaussian $|Z_1|$ on Y (see Eling et al., 2010). The more positive (negative) δ, the more pronounced to the right (left) the skewness. Fig. 1 illustrates the impact of δ on the skewness of the skew-normal distribution (all figures in this paper were generated using a package available for the software R; see http://azzalini.stat.unipd.it/SN/).

An important property of the skew-normal distribution is that all moments exist and are finite. The moment-generating function of $Y \sim SN(\xi, \omega^2, \alpha)$ is given by:

$$M(t) = E\{e^{tY}\} = 2\exp\left(\xi t + \frac{\omega^2 t^2}{2}\right)\Phi(\delta\omega t). \tag{3}$$

Consequently, the moments of Y are easily derived and we obtain easy-to-read expressions for mean, variance, skewness and kurtosis that highlight the influence of the skewness parameter δ:

$$E(Y) = \xi + \omega\sqrt{2/\pi}\,\delta, \tag{4}$$

$$\mathrm{Var}(Y) = \omega^2(1 - 2\delta^2/\pi), \tag{5}$$

$$\mathrm{Skewness}(Y) = (4-\pi)/2(\delta(2/\pi)^{1/2})^3/(1-2\delta^2/\pi)^{3/2}, \tag{6}$$

$$\mathrm{Excess\ Kurtosis}(Y) = 2(\pi-3)(\delta(2/\pi)^{1/2})^4/(1-2\delta^2/\pi)^2. \tag{7}$$

The skew-normal distribution extends the normal distribution in several ways. These can be formalized through a number of properties, such as inclusion (the normal distribution is a skew-normal distribution with shape parameter equal to zero) or affinity (any affine transformation of a skew-normal random vector is skew-normal; for more details, see, e.g., De Luca et al., 2006). Note that the skew-normal distribution can take values of skewness only from −1 to 1. Compared to the normal distribution, it thus extends the range of available skewness, but the range of potential skewness values is still limited.

2.2. Skew-student

The skew-student distribution allows regulating both the skewness and kurtosis of a distribution. This attribute is particularly useful in empirical applications where we want to consider distributions with higher kurtosis than the normal, which is often the case in both finance and insurance applications. One limitation of the skew-normal distribution described in Eq. (1) is that it has a kurtosis only slightly higher than the normal distribution (the maximum excess kurtosis is 0.87). An appealing alternative is offered by a skewed version of the Student's t distribution, introduced by Branco and Dey (2001) and further developed by Azzalini and Capitanio (2003). We define the standardized Student's t skewed distribution using the transformation:

$$X = \frac{Z}{\sqrt{W/\nu}}, \tag{8}$$

where $W \sim \chi^2(\nu)$, with ν degrees of freedom and Z is an independent $SN(0, 1, \alpha)$, instead of $N(0, 1)$ as used to produce the standard t. The linear transformation $Y = \xi + \omega X$ has a *skew-t distribution with parameters* (ξ, ω, α) and we write $Y \sim ST(\xi, \omega^2, \alpha)$. Mean and variance of $Y \sim ST(\xi, \omega^2, \alpha)$ can be computed as follows (see Azzalini and Capitanio, 2003, also for higher moments):

$$E(Y) = \xi + \omega\eta\delta, \quad \text{with } \nu > 1, \tag{9}$$

$$\mathrm{Var}(Y) = \omega^2\left(\frac{\nu}{\nu-2} - \eta\delta^2\right),$$

$$\text{where } \eta = \sqrt{\frac{\nu}{\pi}}\frac{\Gamma\left(\frac{1}{2}(\nu-1)\right)}{\Gamma\left(\frac{1}{2}\nu\right)}. \tag{10}$$

Comparable to the skew-normal case, Eqs. (9) and (10) highlight the influence of δ on the mean and variance of the skew-student