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Maximizing the utility of consumption with commutable life annuities

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A B S T R A C T

The purpose of this paper is to reveal the relation between commutability of life annuities and retirees' willingness to annuitize. To this end, we assume the existence of commutable life annuities, whose surrender charge is a proportion of their actuarial value. We model a retiree as a utility-maximizing economic agent who can invest in a financial market with a risky and a riskless asset and who can purchase or surrender commutable life annuities. We define the wealth of an individual as the total value of her risky and riskless assets, which is required to be non-negative during her lifetime. We exclude the actuarial value of her annuity income in calculating wealth; therefore, we do not allow the individual to borrow from her future annuity income because this income is contingent on her being alive.

We solve this incomplete-market utility maximization problem via duality arguments and obtain semi-analytical solutions. We find that the optimal annuitization strategy depends on the size of proportional surrender charge, with lower proportional surrender charges leading to more annuitization. We also find that full annuitization is optimal when there is no surrender charge or when the retiree is very risk averse. Surprisingly, we find that in the case for which the proportional surrender charge is larger than a critical value, it is optimal for the retiree to behave as if annuities are *not* commutable. We provide numerical examples to illustrate our results.

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1. Introduction and motivation

As a financial product designed for hedging lifetime uncertainty, a life annuity is a contract between an annuitant and an insurance company. For a single premium immediate annuity (SPIA), in exchange for a lump sum payment, the company guarantees to pay the annuitant a fix amount of money periodically until her death. Optimal investment problems in a market with life annuities have been extensively studied since the seminal paper of Yaari (1965); see, for example, the references in Milevsky and Young (2007). With the assumption that there are only bonds and annuities in the financial market, Yaari (1965), as well as Davidoff et al. (2005) among others, prove that it is optimal for an individual with no bequest motive to fully annuitize. In reality, the volume of voluntary purchases by retirees is much lower than predicted by such models, which is the so-called "annuity puzzle".

One well-explored reason for retirees' reluctance to annuitize is that they wish to retain their wealth in liquid form so that they can leave it to their heirs. Davidoff et al. (2005) show that if annuities are priced fairly, then people set aside what they wish to bequeath and annuitize the remainder of their wealth. Lockwood (2012) show that modest bequest motives can severely reduce or eliminate annuity purchasing. He finds that bequest motives that have little effect on saving or on optimal purchasing of actuarially *fair* annuities can have a large effect on the demand for actuarially *unfair* annuities. In reality, annuities are priced unfairly due to loads for risk and administrative costs, so Lockwood's work offers an excellent explanation for the annuity puzzle. Another explanation for the annuity puzzle lies in retirees' fear that issuers of annuities may default. Jang et al. (2009) show that this fear, indeed, affects the demand for annuities. Finally, Benartzi et al. (2011) argue that framing issues might have more of an impact on annuitization than liquidity and irreversibility; also, see the many references therein.

According to a recent survey in the United Kingdom by Gardner and Wadsworth (2004), over half of the individuals in the sample chose not to annuitize given the option. The dominant reason given for not wanting to annuitize is the preference for flexibility. It is well known that annuity income is *not* commutable. Annuity holders can neither surrender for a refund nor short-sell (borrow against) their purchased annuities, even when they are in urgent need of money. However, if life annuities were commutable and, thus, more flexible, we expect that retirees would purchase more annuities. Our work is motivated by the potential relation between commutability of life annuities, specifically SPIAs, and retirees' willingness to annuitize.

In this paper, we investigate how commutability of life annuities affects annuitization, consumption, and investment strategies of a retiree. To this end, we assume the existence of a

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market of *commutable life annuities*, a riskless asset (bond or money market), and a risky asset (stock), and we focus on how including commutable life annuities encourages more annuitization. The commutable annuity, which is a SPIA with a surrender option, has both a purchase price and a surrender value. We assume that the purchase price of this commutable annuity is equal to the expected present value of future payments to the annuity holder, the so-called *actuarial present value*. The surrender value is the actuarial present value less a proportional surrender charge (denoted by p). A retiree is allowed to purchase additional annuity income or to surrender her existing annuity income, and she can invest in the riskless and risky assets in the market as well.

To model the behavior of a utility-maximizing retiree in such a financial market, we formulate a continuous-time optimal consumption and asset allocation problem. We assume that the utility function of the retiree exhibits constant relative risk aversion (CRRA), and we determine the optimal strategy that maximizes the retiree's expected discounted utility of lifetime consumption. We are especially interested in the relation between the optimal annuitization strategy and the size of the proportional surrender charge, the factor that determines the financial flexibility of life annuities.

Our model is an extension of the classical asset allocation framework of Merton (1971). Merton considers the problem of optimal consumption and investment in a complete market with a riskless asset and a risky asset. Cox and Huang (1989) first extend the model to the case of an incomplete market. He and Pagès (1993) consider the case with the presence of labor income. Koo (1998) considers the case in which labor income is subject to uninsurable risk and a liquidity constraint. Davis and Norman (1990) extend the model to an imperfect market in which buying and selling of the risky asset is subject to proportional transaction costs. Øksendal and Sulem (2002) consider the case with the presence of both fixed and proportional transaction costs. See also Elie and Touzi (2008), Karatzas et al. (1997), Tahar et al. (2005), and Egami and Iwaki (2009) for other extensions. The problem treated in our paper is a direct generalization of the one in Milevsky and Young (2007), in which the life annuity is not commutable.

Mathematically, our work is closely related to the literature on optimal investment under proportional transaction costs, as in Davis and Norman (1990), Shreve and Soner (1994), and the more recent survey by Cadenillas (2000). Because the surrender value is proportional to the actuarial present value of the annuity, there is a proportional transaction cost associated with selling or surrendering annuities, although we assume there is no corresponding transaction cost in buying annuities. The optimal investment strategy in Davis and Norman (1990) is one of singular and impulse control; if stock and bond holdings initially lie outside a given "wedge", then the investor immediately buys or sells shares of stock to reach the wedge (impulse control) and afterwards buys or sells instantaneously to remain within that wedge (singular control). We find that the resulting optimal annuitization strategy in our model is of a similar form. Indeed, if wealth and annuity income lie outside a given linearly defined region, then the retiree immediately buys annuity income to reach the region via impulse control and afterwards invests, consumes, and annuitizes to remain within that region via singular control.

Our work is also related to that of liquidity constraints in the presence of (labor) income because we do not allow the individual to borrow against future annuity income, that is, (liquid) wealth must remain non-negative at all times. He and Pagès (1993) consider the problem of maximizing utility of consumption for an individual with stochastic income under borrowing constraints, which they solved via a duality method, similar to the one in this paper. Duffie et al. (1997) consider a similar problem, but generalized the stochastic income process such that its randomness was not spanned by assets in the financial market. Because they assumed that preferences exhibit constant relative risk aversion, they were able to reduce the dimension of the state space from two to one, as we do in this paper.

The commutability of annuities in our model complicates the optimal decisions of the retiree. It leads to a two-dimensional optimal control problem in an incomplete market. The optimal strategy depends on two state variables, wealth and existing annuity income. Taking advantage of the homogeneity of the CRRA utility, we simplify our problem to a one-dimensional equivalent problem, whose value function solves a non-linear differential equation. Via the Legendre dual, we linearize this differential equation and, we indirectly solve for the maximized utility and optimal strategies. We prove the optimality of these solutions through a verification theorem. Milevsky et al. (2006) and Milevsky and Young (2007) also apply this duality argument.

We find that when the proportional surrender charge is smaller than a critical value, an individual keeps wealth to one side of a separating ray in wealth–annuity space by purchasing more annuity income. The slope of this ray increases as p decreases; that is, an individual is more willing to annuitize as the proportional surrender charge decreases. When her wealth reaches zero, the individual continues to invest in the risky asset by borrowing from the riskless account and surrenders annuity income to keep her wealth non-negative, as needed.

By contrast, when the proportional surrender charge is larger than this critical value, the retiree does not invest in the risky asset when her wealth is zero. Additionally, she does not surrender her annuity income; instead, she reduces her consumption to a rate lower than her annuity income in order to accumulate wealth. For comparison's sake, we mention that Duffie et al. (1997) show when wealth reaches zero in their problem with stochastic labor income, the individual's investment in the risky asset is zero and consumption occurs at a rate less than income. More surprisingly, we find that in the case when the surrender charge is larger than the critical value, the optimal annuitization, investment, and consumption strategies do *not* depend on the size of the surrender charge. An individual behaves as if the annuity is not commutable and does not surrender existing annuity income under any circumstances. We use a variety of numerical examples to illustrate our results.

The remainder of this paper is organized as follows. In Section 2, we present the financial market in which the individual invests her wealth. In addition to investing in riskless and risky assets, the individual can purchase or surrender commutable life annuities. In Section 3.1, we consider two special cases: $p = 0$ and $p = 1$. We solve the case $p = 0$ in the primal space by connecting it to a classical Merton problem. By analyzing the retirees' optimal strategies in these two special cases, we gain insight for solving the more general cases. We consider the case when the proportional surrender charge is smaller than some critical value in Section 3.2, and in Section 3.3, we discuss the case when the proportional surrender charge is larger than some critical value. We present properties of the optimal strategies in Section 4 both analytically and numerically. Section 5 concludes our paper.

2. Problem formulation

In this section, we first introduce the assets in the financial and annuity markets: a riskless asset (bond or money market account), a risky asset (stock), and commutable life annuities. Then, we define the maximized utility function, which is the objective function for our optimal control problem. After that, we preliminarily discuss a retiree's optimal strategy. Finally, we prove a verification theorem, which we will use to validate our solution in the next section.

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