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Modeling credit value adjustment with downgrade-triggered termination clause using a ruin theoretic approach

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ABSTRACT

Downgrade-triggered termination clause is a recent innovation in credit risk management to control counterparty credit risk. It allows one party of an over-the-counter derivative to close off its position at marked-to-market price when the other party's credit rating downgrades to an agreed alarming level. Although the default risk is significantly reduced, the non-defaulting party may still suffer losses in case that the other party defaults without triggering the termination clause prior to default. At the heart of the valuation of credit risk adjustment(CVA) is the computation of the probability of default. We employ techniques from ruin theory and complex analysis to provide solutions for probabilities of default, which in turn lead to very efficient and accurate algorithms for computing CVA. The underlying risk model in question is an extension of the commercially available KMV–Merton model and hence can be easily implemented. We provide a hypothetical example of CVA computation for an interest-rate swap with downgrade-triggered termination clause. The paper also contributes to ruin theory by presenting some new results on finite-time ruin probabilities in a jump-diffusion risk model.

1. Introduction

The recent financial crisis which started in 2007 with the credit crunch in the US housing market and quickly spread out to nearly every fabric of the global economy has driven financial institutions, regulators and academics around the world to investigate the root causes and to establish new policies, procedures and trading practices to address the imminent problems facing the financial markets. One area under particular scrutiny is the practice of the banking industry on monitoring and managing counterparty credit risk, which has caused a devastating rippling effect on other sectors of the global economy.

Counterparty credit risk refers to the risk of financial losses to one party of a bilateral financial arrangement when the other party fails to fulfill its contract obligation. The financial industry has traditionally controlled counterparty credit risk by setting credit limit policies and requiring collateral on credit exposures. Thus derivative traders have to forego trading opportunities with credit exposure that exceeds a prescribed credit limit. Nevertheless, numerous examples from the 2007 crisis have shown that even the high profile institutions rated with highest credit ratings by nearly all major rating agencies could suddenly go bankrupt or otherwise suffer severe crippling losses. As the industry weathers through the crisis, many have realized that counterparty credit risk is an inherited risk in trading with each other and the cost of such risk should be reflected appropriately on their derivative books. Among many other changes in practice, many financial institutions have seen transitions from traditional credit limit policies to new accounting standards and procedures for the implementation of Credit Value Adjustment (CVA), which in essence puts a price on the counterparty credit risk. The transition is driven by many factors. (1) Changes in regulation. For instance, Financial Accounting Statement (FAS) 157 sets guidelines for how enterprise must report market or fair value and require them to account for expected losses associated with counterparty defaults. (2) Improved trading practice. Many institutions that have implemented fair value accounting procedures charge CVA from trading departments for credit exposure and hence provide incentives for traders to monitor and manage their overall credit risk. Similar in mechanism to self-insurance, the CVA also provides a buffer to absorb losses from potential counter-party default.

With the increasing complexity of product development in equity-based insurance products, more and more insurance companies entered over-the-counter trading with other financial institutions, inevitably exposing themselves to counterparty credit risk. Although the actuarial profession is well-known for expertise in quantifying, assessing and managing a wide variety of risks associated with the insurance business, there is relatively scarce research work in the actuarial literature on the issue of credit

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risk modeling. Hence we attempt to initiate a discussion on the modeling and valuation of counterparty credit risk using actuarial techniques.

1.1. Credit value adjustment

Counterparty credit risk is similar to many traditional insurable risks in that losses are contingent on random events. However, there are at least two main features of the counterparty credit risk that set it apart from other insurable risks and for which the classical severity-frequency models are not immediately applicable. (1) The risk exposure evolves over time due to the nature of financial derivatives. (2) The losses in the event of counterparty default are also uncertain. The portfolio/asset in the agreement is often marked-to-market at the time of default, which may divert far away from book values. Therefore, the time-varying risk exposure, uncertain loss at default as well as the likelihood of default are all factored into the valuation of CVA in the banking industry.

In a simplified formula, the CVA is often quoted as the expected value of possible losses throughout the term of the arrangement, which can be estimated by the product of (1) loss given default (LGD), which is a percentage of loss due to counterparty default, (2) potential future exposure (PFE), which measures the total value of exposure on each valuation date, and (3) probability of counterparty default (PD) for each valuation period,

$$\mathrm{CVA} = \mathrm{LGD} \times \sum_{i=1}^{n} \Big[\mathrm{PFE}(t_i) \times \mathrm{PD}(t_{i-1}, t_i) \Big], \tag{1.1}$$

where $\mathrm{PFE}(t)$ is the total value of exposure on the valuation date $t = t_0, \ldots, t_n$, and $\mathrm{PD}(s, t)$ is the probability of counterparty default between dates s and t. Interested readers may consult Canabarro and Duffie (2003) and Crouhy et al. (2001) for a variety of models used by banks and regulators to quantify and model counterparty credit risk.

Among all three components used for CVA modeling, the first two factors, namely LGD and PFE, are usually easier to measure or estimate based on market information. The LGD is often assumed in the literature to be a fixed ratio based on specific information on the nature of counter-party transactions. However, if necessary, the randomness in LGD can be accommodated in the simulation of PFE.

According to De Prisco and Rosen (2005), the most prevailing method in market practice of determining the PFE is to compute the distribution of future exposure on OTC derivatives on a set of valuation dates ($\mathrm{PFE}(t)$, $t = t_0, \ldots, t_n$) in four steps: (1) *Scenario Generating.* As the payoffs of derivative products are often dependent on cash flows between the involved parties, it is the first and foremost task to generate all possible market scenarios of their trading positions. Each market scenario is a realization of a set of price factors that affect the values of trades in the portfolio. (2) *Instrument Valuation.* Every financial instrument involved is valued at the contractual level for every scenario generated and at every valuation date. (3) *Portfolio Aggregation.* Since financial institutions often use credit risk mitigation techniques such as requiring each other to post collateral when the uncollateralized exposure exceeds a threshold, the effect of these provisions should considered for each scenario generated. (4) *Statistics Calculation.* The realizations of exposures on all possible scenarios are computed by aggregating all transactions with a counterparty and hence produce an empirical distribution of exposure at the counterparty level. Depending on their own practice, the institutions may choose to compute different statistics for risk monitoring and management.

The most difficult task in the valuation of CVA appears to be the determination of default probabilities. Unlike the loss distribution of other insurable risks, the probability of default for a particular counterparty may not be directly estimated from historical default rates of other firms. Even if one is willing to believe that actual default rates are equal to historical averages, there is often lack of sufficient data on default events for firms of comparable size, debt structure or exposure to similar risks, in order to make any credible estimation. Many have questioned the reliability of historical default data on companies categorized by credit ratings, which are usually defined on a qualitative scale. Readers are recommended to read Chapter 9 of Crouhy et al. (2001) for more details.

This technical difficulty has given rise to a vast amount of research work in financial literature on the modeling of default probabilities. We can roughly group the mainstream models into three categories: (1) Structural Models. The models often propose the asset and liability structure of the counterparty and the event of default is viewed as the first passage time of asset process down-crossing the liability level. Examples of structural models for the valuation of contingent claims and bankruptcy rates can be found in Black and Cox (1976), Leland (1994) and Leland and Toft (1996), etc. There are also many empirical studies on a collection of structural models for default probabilities, such as Duffie and Singleton (1997) and Huang and Huang (2003), etc. (2) Intensity-Based/Reduced-Form Models. In contrast with the structural models, the intensity-based models regard default as an exogenous random event characterized by a deterministic default intensity function or more generally a stochastic intensity process. (3) Empirical Extraction. Assuming that market prices reflect investors' perception of default rates, the probabilities can be extracted from the term structure of credit-default swap (CDS) spreads, which are directly observable in CDS markets. Interested readers can read Bielecki and Rutkowski (2002) for a comprehensive account of both structural and reduced-form models and Yi (2010) for examples of empirical extraction methods used in the banking industry.

1.2. Credit value adjustment subject to ATE

In the wake of the 2007 credit crisis, new mechanisms for credit risk management have also been developed in the banking industry such as the alternative termination event (ATE) clause which allows investors to close out their positions at market value prior to maturity under certain pre-described circumstances. Among many other specific forms, one type of ATE clause that has been increasingly popular is the downgrade-triggered termination clause, under which one party of the contract may choose to terminate if the credit rating of the other party drops to or below an agreed threshold. Without the protection of such a clause, the credit quality of the counterparty may continue to worsen, eventually leading to default and causing losses to the non-defaulting party. However, the downgrade clause is not a panacea for all counterparty risks. Losses may incur when the counterparty defaults without ever triggering the clause. Hence the underwriting of such clauses complicates the valuation of the CVA and poses a new technical challenge. Information on the downgrade-triggered termination clause can be found in Carver (2011). Throughout the rest of the paper, we shall call it a downgrade trigger for short.

Despite its significance in credit risk management, there have only been very few papers on the valuation of the CVA with downgrade trigger, probably attributable to its fairly short history. It is clear that the method of extracting probabilities of default from CDS spreads is not suitable for modeling CVA with downgrade trigger, since the prices of CDS do not provide adequate information on downgrades. Yi (2010) was among the