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Lévy risk model with two-sided jumps and a barrier dividend strategy

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ABSTRACT

In this paper, we consider a general Lévy risk model with two-sided jumps and a constant dividend barrier. We connect the ruin problem of the ex-dividend risk process with the first passage problem of the Lévy process reflected at its running maximum. We prove that if the positive jumps of the risk model form a compound Poisson process and the remaining part is a spectrally negative Lévy process with unbounded variation, the Laplace transform (as a function of the initial surplus) of the upward entrance time of the reflected (at the running infimum) Lévy process exhibits the smooth pasting property at the reflecting barrier. When the surplus process is described by a double exponential jump diffusion in the absence of dividend payment, we derive some explicit expressions for the Laplace transform of the ruin time, the distribution of the deficit at ruin, and the total expected discounted dividends. Numerical experiments concerning the optimal barrier strategy are performed and new empirical findings are presented.

1. Introduction

Since the pioneering work by De Finetti (1957), the problem of finding the optimal dividend-payment strategy has been studied extensively. De Finetti found that, if the goal is to maximize the expected discounted dividends, the optimal strategy must be a barrier strategy. Some related works on this subject include, among others, Gerber and Shiu (1998, 2004), Siegl and Tichy (1999), Højgaard (2002), Irbäck (2003), Sheldon et al. (2003), Zhou (2005), Kyprianou and Palmowski (2007), Renaud and Zhou (2007), Belhaj (2010). Recently, based on the fluctuation theory of spectrally negative Lévy processes, Avram et al. (2007), Loeffen (2008), Loeffen (2009) have studied the optimal dividend problem for general spectrally negative Lévy processes, and provided sufficient conditions under which the barrier strategy solves the de Finetti optimal dividend problem. Kyprianou et al. (2010) further generalized the results in Avram et al. (2007) and Loeffen (2008, 2009) by showing that if the Lévy measure of the spectrally negative Lévy process has a log convex density, the barrier strategy is optimal. Loeffen and Renaud (2010) finally proved that the results on optimal dividend strategies of these papers are still valid when the corresponding condition imposed on the Lévy measure is replaced by the condition that the tail of the Lévy measure is

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log-convex. All of the above mentioned work is based on spectrally one-sided models.

Recently, risk models with two-sided jumps attract more and more attention. In this kind of models, the upward jumps can be interpreted as random returns (obtained by investing the initial asset and the insurance premium) of an insurance company, while the downward jumps are interpreted as random losses (from investment or claim indemnity) of the company. For research on this kind of models, we refer, among others, to Perry et al. (2002), Cai and Yang (2005), Jacobsen (2005), Xing et al. (2008), Cai et al. (2009), Zhang et al. (2010), Chi (2010) and Albrecher et al. (2010). Most recently, Chi and Lin (2011) studied the threshold dividend strategy when the risk process follows a Lévy process with two-sided jumps.

The current paper aims at studying risk models with two-sided jumps and a (constant) barrier dividend strategy. To the best of our knowledge, Paulsen and Gjessing (1997), Yin and Yuen (2011) and Yuen and Yin (2011) are the only papers that addressed the barrier dividend problem with two-sided jumps. However, the proofs in these three papers are questionable. It is worthwhile to note that when the surplus process can jump upward, the threshold dividend strategy is drastically different from the barrier dividend strategy, since the former generates a continuous dividend process, while the latter creates a discontinuous one. Thus the techniques used in Chi and Lin (2011) for studying the threshold dividend strategy are not feasible for the barrier dividend strategy in this paper.

The purpose of this paper is to establish some easily implementable results on a general Lévy risk model with two-sided jumps and a barrier dividend strategy. More specifically, we are going to show that the constant barrier dividend problem can be explicitly solved as well for some spectrally two-sided Lévy processes. We first relate the ruin problem of the risk model with barrier dividend strategy to the first passage problem of the risk model reflected at its running maximum (see Proposition 2.1). For a general Lévy risk model, we show that the expected discounted dividend can be expressed in terms of the Laplace transform of the upward entrance time of the unconstrained Lévy process and the joint Laplace transform of the upward entrance time and the overshoot of the Lévy process reflected at its running maximum (see Theorem 2.1). If the Lévy risk model can be decomposed into two parts, namely a spectrally negative Lévy process with unbounded variation and a subordinator, we can prove that the Laplace transform (as a function of the initial surplus) of the upward entrance time of the Lévy process reflected at its running infimum possesses the smooth pasting property at the reflecting barrier 0. A more general version of this result is proved when the underlying risk model is the so-called double exponential jump diffusion (see, e.g. Kou and Wang, 2003, 2004). The smooth pasting property will serve as a homogeneous Neumann boundary condition when we solve the Feynman–Kac integro-differential equations corresponding to the first passage problems of the reflected Lévy processes. We then study the (joint) Laplace transform of the upward entrance time and the overshoot for the double exponential jump diffusion reflected at its running infimum and maximum, respectively. Then, applying our results above, we find some explicit expressions for the Laplace transform of the time of ruin, the distribution of the deficit at ruin and the expected discounted dividends up to ruin. All our results on the ruin problem are expressed in terms of the parameters of the jump size and the solutions to the Cramér–Lundberg equation corresponding to the underlying double exponential jump diffusion or its dual. A nice feature of our results is that they are explicit functions of the initial surplus and the barrier parameter, which is very handy when we want to solve the optimal dividend barrier. Finally, we present some numerical experiments related to the optimal barrier strategy. The most important empirical finding is that the optimal dividend barrier would depend on the initial surplus if the initial surplus is less than some critical value (in our experiment, the optimal barrier decreases to a positive value as the initial surplus decreases to zero); whereas if the initial surplus is greater than or equal to the critical value, the optimal dividend barrier will be equal to the critical value. The dependence on the initial surplus of the optimal dividend barrier is different from the case in the spectrally negative setting (see, e.g., Gerber and Shiu (1998, 2004) and Kyprianou et al. (2010)), which is due to the incorporation of two-sided jumps in the risk model. Note that a barrier strategy that depends on the initial surplus cannot be optimal among all admissible dividend strategies.

The dividend process corresponding to the barrier strategy is exactly the so-called regulator (or the local time) at the dividend barrier, which can be given as the solution to a Skorokhod problem (see, e.g., Skorokhod (1961), Harrison (1985), Doney and Maller (2007) and Asmussen and Pihlsgard (2007)). The post-dividend surplus process is the so-called reflected jump-diffusion (or jump-diffusion with reflecting barrier). Some theoretical results related to reflected spectrally one-sided Lévy processes can be found in Pistorius (2003, 2004). Kella and Whitt (1992) and Nguyen-Ngoc and Yor (2005) introduced some useful martingales related to the reflected Lévy processes, which are very powerful in various applications in queueing, finance and insurance theory.

Our paper is organized as follows. Section 2 investigates the general Lévy risk model. In particular, we will establish some key identities between the ruin problem and the first passage problem of a Lévy process reflected at its running maximum. In Section 3, we concentrate on the double exponential jump diffusion as a solvable example. Therein we will derive some explicit expressions for the Laplace transform of the time of ruin, the distribution of the deficit at ruin, and the expected discounted dividends. Section 4 provides numerical results on optimal dividend strategy. Section 5 concludes the paper and discusses some potential further research. Some proofs are given in the Appendix.

2. General Lévy risk model with two-sided jumps

Let $X = \{X_t, t \geq 0\}$ be a Lévy process on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$, where $\{\mathcal{F}_t\}_{t\geq 0}$ satisfies the usual conditions of right-continuity and completeness. Let σ and Π be the Gaussian coefficient and the Lévy measure of X respectively. When X is of bounded variation, we will write $X_t = dt + J_t$ where d is the drift and J is a pure jump Lévy process. Throughout this paper, we assume that either X has unbounded variation or Π is absolutely continuous with respect to the Lebesgue measure, i.e.,

$$\sigma > 0 \quad \text{or} \int_{|x|<1} |x|\Pi(dx) = \infty \text{ or } \Pi(dx) \ll dx. \tag{2.1}$$

Denote by $\mathbb{P}_x, x \in \mathbb{R}$ the law of $X + x$ under $\mathbb{P}$. Let $\mathbb{E}_x$ be the expectation operator corresponding to $\mathbb{P}_x$. Let T_a^+ and T_a^- be the entrance times of the Lévy process X into $(a, +\infty)$ and $(-\infty, -a)$, respectively:

$$T_a^+ = \inf\{t \geq 0: X_t > a\}, \qquad T_a^- = \inf\{t \geq 0: X_t < -a\}, \tag{2.2}$$

with the convention $\inf \emptyset = \infty$. Define $Y := X - I$ and $\hat{Y} = S - X$ as the Lévy process X reflected at its running infimum I and at its running supremum S, respectively:

$$I_t := \inf_{s\leq t}(X_s \wedge 0), \qquad S_t = \sup_{s\leq t}(X_s \vee 0). \tag{2.3}$$

Let $\hat{X} = -X$ be the dual process of X, we have

$$\begin{aligned} S_t &= \sup_{0\leq s\leq t}(X_s \vee 0) = -\inf_{0\leq s\leq t}((-X_s) \wedge 0) \\ &= -\inf_{0\leq s\leq t}(\hat{X}_s \wedge 0) =: -\hat{I}, \end{aligned} \tag{2.4}$$