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Statistical analysis of model risk concerning temperature residuals and its impact on pricing weather derivatives[☆]

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ABSTRACT

In this paper we model the daily average temperature via an extended version of the standard Ornstein Uhlenbeck process driven by a Levy noise with seasonally adjusted asymmetric ARCH process for volatility. More precisely, we model the disturbances with the Normal inverse Gaussian (NIG) and Variance gamma (VG) distribution. Besides modelling the residuals we also compare the prices of January 2010 out of the money call and put options for two of the Slovenian largest cities Ljubljana and Maribor under normally distributed disturbances and NIG and VG distributed disturbances. The results of our numerical analysis demonstrate that the normal model fails to capture adequately tail risk, and consequently significantly misprices out of the money options. On the other hand prices obtained using NIG and VG distributed disturbances fit well to the results obtained by bootstrapping the residuals. Thus one should take extreme care in choosing the appropriate statistical model.

1. Introduction

The recent financial crisis has primarily exposed the problem of model risk and the inability of many models to capture low frequency, high severity events. It appears that most models used in finance tend to ignore tail events which disproportionately contribute to huge financial losses and insolvency. In calmer times when today's events are "sampled" from the centre of the distribution function the errors of models do not become evident and financial institutions thrive on the models they use for their ongoing business. It is the occurrence of tail events that raises doubts about the financial models used.

Weather derivatives are still a fairly small but a growing part of the derivatives business. Weather is impacted by nature and one may speculate that model risk is much smaller than in the case of stock returns that are affected by humans. Although stock returns are generally more leptokurtic than temperature, we argue in this paper that model risk cannot be completely ignored. With the growing interest in weather derivatives we are seeing also stronger academic interest in the subject of pricing weather derivatives. In the past a common assumption regarding deviations from expected temperature was that of normally distributed disturbances. Despite evidence to the contrary, this assumption is still fairly widely used (Alaton et al., 2002; Mraoua and Bari, 2007; Campbell and Diebold, 2005; Davis, 2001; Taylor and Buizza, 2006; Härdle and López, 2009; Jewson and Brix, 2005; Huang et al., 2008). Only recently some authors have (Viel and Connor (2010), Zapranis and Alexandridis (2009), Benth and Saltyte Benth (2005a)) questioned the validity of this assumption and proposed an alternative specification for the distribution of temperature and its residuals. From the viewpoint of the seller it is crucial to know the dynamics of the temperature as well as possible. It is thus vital to try and upgrade models so that the distribution of future payoffs can be determined with the smallest margin of error.

The market of weather derivatives is an example of an incomplete market. Therefore, one has to use one of the possible approaches available such as an Esscher transform or mean correcting the exponential of a Levy process. Nevertheless, the problem of pricing weather derivatives is linked to determining the statistical properties of the weather indices as accurately as possible. As mentioned, most of the methodology still relies on a normal distribution which is adequate for pricing in-the-money options but inappropriate for pricing out-of-the-money options that rely heavily on an accurate estimation of the tails of the distribution function.

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In this paper we address the current situation by first extending the approach of modelling temperature by including monthly temperature trends and using the asymmetric ARCH to model volatility. In the context of temperature modelling we use an extension of the standard Ornstein Uhlenbeck process driven by Levy noise with a seasonally adjusted asymmetric ARCH process for volatility. More precisely, we model the disturbances with a normal-inverse Gaussian ('NIG') and variance-gamma ('VG') distribution. In the second part of the paper we apply the Esscher transform to calculate the prices of January out-of-the-money options for normal NIG and VG distributed disturbances. The prices thereby obtained are compared to prices obtained by bootstrapping the residuals. This allows us to obtain a rough estimate of the model risk involved when employing the standard approach of modelling the residuals via a normal distribution.

The results of our temperature model reveal that most of the monthly dummies are significant, whereas the standard deviation of noise is not only affected by squared past residuals but also by the residuals themselves. Namely, periods of high volatility are much more likely to come in periods of above-average temperature than in the periods of below-average temperature. The results of our numerical analysis of pricing out-of-the-money options using a different statistical model for residuals indicate the normal model fails to adequately capture tail risk and consequently significantly misprices out-of-the-money options. Thus, one should take extreme care when choosing the appropriate statistical model.

This rest of the paper is organised as follows. In Section 2 we give a short description of infinitely divisible distributions with an emphasis on VG and NIG distributions. A description of the temperature model together with the model characteristics is presented in Section 3. This section also contains the regression results for Ljubljana and Maribor data using the aforementioned model to explain the dynamics of temperature. In Section 4 we outline the details of our pricing mechanism. Section 5 outlines the numerical results. Concluding remarks are made in Section 6.

2. Infinitely divisible distributions

In this section we briefly overview the distributions used in the paper for modelling the residuals within the temperature model. We consider NIG and VG distributions which belong to the class of infinitely divisible distributions. The class of infinitely divisible distributions can be specified via their characteristic function (Schoutens, 2003).

2.1. Variance gamma (VG) distribution

A random variable X is VG distributed with parameters $\mu, \theta \in \mathbb{R}$ and $\sigma, \nu > 0$ if the characteristic function is given by

$$\phi_X(z) = E[e^{izX}] = e^{i\mu z}\left(1 - i\theta\nu z + \frac{1}{2}\nu\sigma^2 z^2\right)^{-\frac{1}{\nu}}, \quad z \in \mathbb{R}. \tag{1}$$

As shown in Moosbrucker (2006) the density of VG distribution function equals

$$f_X(x) = \frac{2\exp\left(\frac{\theta x}{\sigma^2}\right)}{\nu^{\frac{1}{\nu}}\sqrt{2\pi}\sigma\Gamma\left(\frac{1}{\nu}\right)}\left(\frac{(x-\mu)^2}{\frac{2\sigma^2}{\nu}+\theta^2}\right)^{\frac{1}{2\nu}-\frac{1}{4}} \times K_{\frac{1}{\nu}-\frac{1}{2}}\left(\frac{1}{\sigma^2}\sqrt{(x-\mu)^2\left(\frac{2\sigma^2}{\nu}+\theta^2\right)}\right) \tag{2}$$

where K_ν is a modified Bessel function of the second kind

$$K_\nu(x) = \frac{1}{2}\int_0^\infty y^{\nu-1}\exp\left(-\frac{1}{2}x(y+y^{-1})\right)dy. \tag{3}$$

The first four moments of the VG can be inferred from

$$E(x) = \mu + \theta \tag{4}$$

$$\mathrm{Var}(x) = \nu\theta^2 + \sigma^2 \tag{5}$$

$$Skew(x) = \nu\theta\frac{3\sigma^2 + 2\nu\theta^2}{(\sigma^2+\nu\theta^2)^{3/2}} \tag{6}$$

$$Kurt(x) = 3\left(1 + 2\nu - \nu\sigma^4(\sigma^2+\nu\theta^2)^{-2}\right). \tag{7}$$

2.2. Normal inverse Gaussian (NIG) distribution

The Normal inverse Gaussian distribution is a four parameter $(\mu, \alpha, \beta, \delta)$ distribution with characteristic function $\phi_X(z)$

$$\phi_X(z) = E[e^{izX}] = e^{i\mu z + \delta\left(\gamma - \sqrt{\alpha^2-(\beta+iz)^2}\right)}, \quad z \in \mathbb{R} \tag{8}$$

with $\gamma = \sqrt{\alpha^2 - \beta^2}$.

The density of NIG distribution function

$$f_X(x) = \frac{\alpha\delta\cdot\exp(\delta\gamma + \beta(x-\mu))}{\pi\sqrt{\delta^2+(x-\mu)^2}}K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right) \tag{9}$$

where K_1 is a modified Bessel function of the second kind

$$K_\nu(x) = \frac{1}{2}\int_0^\infty y^{\nu-1}\exp\left(-\frac{1}{2}x(y+y^{-1})\right)dy. \tag{10}$$

The first four moments of the NIG distribution can be inferred from

$$E(x) = \mu + \delta\frac{\beta}{\gamma} \tag{11}$$

$$\mathrm{Var}(x) = \delta\frac{\alpha^2}{\gamma^3} \tag{12}$$

$$Skew(x) = 3\frac{\beta}{\alpha\sqrt{\delta\gamma}} \tag{13}$$

$$Kurt(x) = 3 + 3\left(1 + 4\left(\frac{\beta}{\alpha}\right)^2\right)\frac{1}{\delta\gamma}. \tag{14}$$

3. Time-series temperature modelling

As mentioned we extend the standard Ornstein Uhlenbeck type process driven by a Levy noise. We propose to model the time dynamics of temperature as follows:

$$dT(t) = \kappa(T(t) - s(t) - m(t))dt + \zeta f(T'(t), T''(t))dt + ds + \sigma(t)dL(t). \tag{15}$$

Here $T(t)$ stands for temperature at time t, κ is a mean reversion coefficient, $s(t)$ is the seasonally adjusted temperature, $f(T'(t), T''(t))$ is a linear function capturing the effects of higher order derivatives of temperature, $m(t)$ is the trend in expected temperature, $\sigma(t)$ is the adapted volatility process and $L(t)$ is a Levy process noise term. Observe that our model can be viewed as an extension of Benth and Saltyte Benth (2005a) with an extra term $\zeta f(T'(t), T''(t))$ that allows for more general formulation of temperature dynamics. Discretising the model we obtain the following discrete time analog of the upper equation

$$T(k+1) = T(k) + \kappa(m(k) + s(k) - T(k)) + \sum_{l=1}^{P}\rho(l)T(k-l) + \varepsilon_k \tag{16}$$

where $m(k)$ denotes the expected monthly trend in temperature, and P indicates the number of lags included in the equation. Observe that the upper equation deviates somewhat from the