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An extension of the Wang transform derived from Bühlmann's economic premium principle for insurance risk[☆]

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Abstract

It is well known that the Wang transform [Wang, S.S., 2002. A universal framework for pricing financial and insurance risks. Astin Bull. 32, 213–234] for the pricing of financial and insurance risks is derived from Bühlmann's economic premium principle [Bühlmann, H., 1980. An economic premium principle. Astin Bull. 11, 52–60]. The transform is extended to the multivariate setting by [Kijima M., 2006. A multivariate extension of equilibrium pricing transforms: The multivariate Esscher and Wang transforms for pricing financial and insurance risks, Astin Bull. 36, 269–283]. This paper further extends the results to derive a class of probability transforms that are consistent with Bühlmann's pricing formula. The class of transforms is extended to the multivariate setting by using a Gaussian copula, while the multiperiod extension is also possible within the equilibrium pricing framework.
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1. Introduction

In the actuarial literature, there have been developed many probability transforms for pricing financial and insurance risks. Such methods include the variance loading, the standard deviation loading, and the Esscher transform. Recently, Wang (2000, 2002) proposed a pricing method based on the following transformation from $F(x)$ to $F^*(x)$:

$$F^*(x) = \Phi[\Phi^{-1}(F(x)) + \theta], \tag{1.1}$$

where Φ denotes the standard normal cumulative distribution function (CDF for short) and θ is a constant. The transform is now called the *Wang transform* and produces a risk-adjusted CDF $F^*(x)$. The mean value evaluated under $F^*(x)$ will define a risk-adjusted "fair value" of risk X with CDF $F(x)$ at some future time, which can be discounted to time zero using the risk-free interest rate. The parameter θ is considered to be a risk premium.

The Wang transform not only possesses various desirable properties as a pricing method, but also has a sound economic interpretation. For example, the Wang transform (as well as the Esscher transform) is the only distortion function, among the family of distortions, that can recover CAPM (the capital asset pricing model) for underlying assets and the Black–Scholes formula for options. Also, the transform (1.1) is consistent with Bühlmann's economic premium principle. See Wang (2003) and Kijima (2006) for details.

More precisely, Bühlmann (1980) considered risk exchanges among a set of agents. Each agent is characterized by his/her exponential utility function $u_j(x) = -\mathrm{e}^{-\lambda_j x}$, $j = 1, 2, \ldots, n$, and faces a risk of potential loss X_j. In a pure risk exchange model, Bühlmann (1980) derived the equilibrium pricing formula

$$\pi(X) = E[\eta X], \qquad \eta = \frac{\mathrm{e}^{-\lambda Z}}{E[\mathrm{e}^{-\lambda Z}]}, \tag{1.2}$$

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where $Z = \sum_{j=1}^{n} X_j$ is the aggregate risk and λ is given by

$$\lambda^{-1} = \sum_{j=1}^{n} \lambda_j^{-1}, \qquad \lambda_j > 0.$$

The parameter λ is thought of the *risk aversion index* of the representative agent in the market. Wang (2003) showed that the transform (1.1) can be derived from the equilibrium pricing formula (1.2) under some assumptions on the aggregate risk. The result is extended to the multivariate setting by Kijima (2006).

In this paper, we derive a class of transforms that are consistent with Bühlmann's economic premium principle (1.2), thereby extending the results of Wang (2003). Namely, based on the idea of Kijima (2006), we obtain the transform

$$F^*(x) = E[\Phi(G^{-1}(F(x))Y + \theta)], \tag{1.3}$$

where Y is *any* positive random variable and the expectation is taken with respect to Y. Here, $G(x)$ denotes the CDF of random variable U/Y and U represents a standard normal random variable, independent of Y. In particular, when $Y = 1$ almost surely, we have $G(x) = \Phi(x)$, so that the transform (1.3) is reduced to the Wang transform (1.1). The transform (1.3) can be extended to the multivariate setting by using a Gaussian copula, in order to preserve the linearity for the pricing functional.[1] A multiperiod extension is also possible within the equilibrium pricing framework.

It is often said that a drawback of the Wang transform is the normal CDF Φ appeared in (1.1), that never match the fat-tailness observed in the actual markets. In fact, some empirical studies suggest to use t distributions, whose CDF is denoted by $T_\nu(x)$, with $\nu = 3$ or 4 degrees of freedom for return distributions of financial and insurance assets (see, e.g., Platen and Stahl (2003)). Hence, it is natural to consider the case that $Y = \sqrt{\chi_\nu^2/\nu}$, where χ_ν^2 denotes a chi-square random variable with ν degrees of freedom. As Kijima and Muromachi (2006) observed, this case leads to the two-parameter transformation

$$F^*(x) = P_{\nu:-\lambda}[T_\nu^{-1}(F(x))], \tag{1.4}$$

where $P_{\nu:\delta}$ denotes the CDF of non-central t distribution with ν degrees of freedom and non-centrality parameter δ. However, contrary to our intuition, the risk-adjusted distribution (1.4) derived from t distributions is not fatter in tail parts than the original Wang transform (1.1) that is derived from normal distributions.

The present paper is organized as follows. In the next section, we review the result of Wang (2003) and consider an alternative (and simple) derivation of the Wang transform (1.1). Using the idea presented in Section 2, the general transform (1.3) is derived in Section 3. Of interest is the comparison of tail distributions derived by the general transform. It is shown in Section 4 that the original Wang transform (1.1) produces the fattest tail distribution among the class of transforms (1.3) that are derived from Bühlmann's economic premium principle (1.2). While the result is further extended to a multivariate setting in Section 5, Section 6 states a multiperiod extension of our result within the equilibrium pricing framework. Section 7 concludes the paper.

2. A pure risk exchange economy

Bühlmann (1980) considered a single-period economy for risk exchanges among a set of agents $j = 1, 2, \ldots, n$. Each agent is characterized by an exponential utility function $u_j(x) = -\mathrm{e}^{-\lambda_j x}$, $x \geq 0$, and initial wealth w_j. Suppose that agent j faces a risk of potential loss X_j and is willing to buy/sell a risk exchange Y_j. If agent j is an insurance company, the risk exchange Y_j is thought of the sum of all insurance policies sold by j. While the original risk X_j belongs to agent j, the risk exchange Y_j can be freely bought/sold by the agents in the market. Denoting the price of Y_j by $\pi(Y_j)$, the equilibrium price for this risk exchange economy is characterized by:

(i) For any j, $E[u_j(w_j - X_j + Y_j - \pi(Y_j))]$ is maximized with respect to Y_j, and
(ii) $\sum_{j=1}^{n} Y_j = 0$ for all possible states.

In this setting, Bühlmann (1980) showed that the equilibrium price $\pi(Y)$ for the risk exchange is given by (1.2).

2.1. The Wang transform

Wang (2003) showed that the transform (1.1) can be derived from Bühlmann's economic premium principle (1.2) under the following assumptions on the aggregate risk $Z = \sum_{j=1}^{n} X_j$:

(i) There are so many individual risks X_j in the market that the aggregate risk Z can be approximated by a normal random variable, and
(ii) The correlation coefficient between $Z_0 = (Z - \mu_Z)/\sigma_Z$ and $U = \Phi^{-1}[F(X)]$ is ρ,[2] where $\mu_Z = E[Z]$ and $\sigma_Z^2 = V[Z]$.

Here, $F(x)$ denotes the CDF of a risk X of interest. For the sake of simplicity, it is assumed that $F(x)$ is continuous and strictly increasing in x. The inverse function of $F(x)$ is denoted by $F^{-1}(x)$.

Recall that, if the random vector (Z_0, U) follows a bivariate normal distribution with correlation coefficient ρ, there exists a normal random variable ξ, independent of (Z_0, U), such that $Z_0 = \rho U + \xi$. Hence, from (1.2), it follows that

$$\pi(X) = \frac{E[X\mathrm{e}^{-\theta U}]}{E[\mathrm{e}^{-\theta U}]}, \qquad \theta = \lambda\sigma_Z\rho.$$

[1] The pricing functional π is said to be linear if $\pi(aX + bY) = a\pi(X) + b\pi(Y)$ for all risks X, Y and constants a, b. If it is not linear, arbitrage opportunities are not precluded. See, e.g., Harrison and Kreps (1979) and Kijima (2002) for details.

[2] For any random variable X with continuous CDF $F(x)$, the random variable $\Phi^{-1}[F(X)]$ follows a standard normal distribution. Hence, the second assumption can be stated that the random vector $(F(X), \Phi(Z_0))$ follows a bivariate Gaussian copula with correlation coefficient ρ.

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